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**The experience of using a cognitive acceleration approach with prospective primary teachers in Chile**

Tornero Ochagavia, Bernardita

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# **The experience of using a Cognitive Acceleration approach with prospective primary teachers in Chile**

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**A thesis submitted in partial fulfilment of the requirements for a  
PhD degree at King's College London**

**School of Social Science and Public Policy  
Department of Education and Professional Studies  
King's College London**

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## Abstract

Cognitive Acceleration in Mathematics Education (CAME) programmes have been used successfully for promoting the development of thinking skills in school students for the last 30 years. Given that the approach has had a tremendous impact on the thinking capabilities of participating students, this study explored the experience of using the programme with prospective primary teachers in Chile. Therefore, this study not only looked at the experience of prospective primary teachers during the CAME course as learners, but also examined how they perceived the approach from their perspective as future teachers, as well as how they could transfer the teaching strategies they observed to their future classrooms.

Given the complexity of the phenomenon under study, this research used a mixed methods approach. For this reason, the impact that the CAME course had on prospective teachers' thinking skills was not only approached by using a test that assessed the participants' improvements in these skills, but their learning and teaching experiences were also recorded through qualitative research tools (learning journals, interviews and field notes).

The main findings indicate that, at the end of the CAME course, prospective teachers not only demonstrated higher thinking levels, but also showed positive attitudinal changes towards teaching and learning in general, and towards mathematics in particular. The participants also had increased confidence in their ability to teach mathematics and to promote thinking skills in their students. In terms of the CAME methodology, prospective teachers not only found it novel and motivating, but also commented that dealing with the thinking skills topic during a university course was both unusual and very important for their professional development. This study also showed that, at the end of the CAME course, prospective teachers felt they had developed strategies that could be used in their classrooms in the future.

In this context, the relevance of the study is not only that it described the impact and the positive results of the first experience of using a CAME approach with prospective teachers, but also that some of the conclusions have significant implications for the teaching of thinking skills and the training of primary school teachers.

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## **I. Introduction**

In modern society, knowledge exchange plays a major role, and the globalised era has posed new challenges for current educational systems. Consequently, the educational goals pursued some decades ago need to be changed to accommodate both new and novel ways of improving the schooling process of future generations. A group of scholars (Torff, 2003, Barak and Dori, 2009, Preiss and Sternberg, 2006) has agreed that one of these new educational aims is the development of thinking skills that allow students to find, manage, select, criticise and update their information. For this reason, teachers need to change their focus from teaching the content of subjects to the promotion of complex cognitive skills that encourage students to become independent and active learners.

Since Piaget and Inhelder (1958) developed their cognitive model, it has been traditionally understood that types of thinking that are more complex are usually acquired during adolescence (Anderson, 2003). According to Piaget and Inhelder (1958), more complex types of thinking are primarily characterised by the presence of logical and hypothetical deductive reasoning. In other words, children are no longer able to think only about their real and concrete experiences, but can also consider that which is possible. In this context, children are now capable of drawing conclusions from hypothetical information and not only from facts.

However, the body of research on cognitive development conducted over the last three decades has shown that a significant proportion of secondary school students (i.e. Shemesh et al., 1992, Adey and Shayer, 1994, Valanides, 1996, 1997a, 1997b) and university students (i.e. Niaz, 1985, Reyes, 1987) has not yet developed these abilities. In this context, Shayer and Adey (1981) decided to design a cognitive acceleration (CA) programme for school students that could reverse this situation. Therefore, they created a set of thinking activities that were used once a fortnight instead of ordinary science lessons in order to promote and enhance the students' general thinking abilities, which could then be transferred to other tasks, situations or domains. The underlying assumption of the approach

was that there is a general thinking processor that natural development can accelerate through specific intervention (Adey, 1999).

Throughout the 1980s, Michael Shayer and Philip Adey investigated how well their CA programme worked in a number of schools in England, using a quasi-experimental design. The intervention was considered successful because the students assigned to the experimental condition showed statistically greater cognitive development after the programme than did their control counterparts. In addition, the authors found that the intervention also had a long-term and far-transfer effect. Although the intervention was set in a science context and was conducted by science teachers, students assigned to the experimental condition obtained better results not only in science but also in national mathematics and English tests. Since CASE produced such promising results in science, CA programmes began to be developed for other school subjects (Adhami et al., 1998, Shayer and Adhami, 2003, Adhami et al., 2005), and according to students' ages (Adey et al., 2001a, Adey et al., 2002, Shayer and Adhami, 2003, Adhami et al., 2005) and countries (Iqbal and Shayer, 2000, Mbano, 2003, Endler and Bond, 2008).

Although CA programmes can be considered as a potential solution for the poor development of thinking skills in many schools today, the problem is complex and other factors inhibit or constrain the uptake of CA programmes in schools. One of these is that prospective teachers may not have yet developed these thinking skills (Silverman and Creswell, 1982, Wyatt, 1983, Brownell et al., 1993) and thus may feel uncomfortable or threatened when trying to work with activities that they themselves found challenging. Thinking skills have also not had a strong presence in most schools or subject curricula, as the emphasis has been on content knowledge rather than on skills. This has led to most teachers that are already working in school classrooms having a very limited idea about the meaning of thinking skills and such skills could be promoted in the classroom, with just a small group of them claiming that promoting thinking is an important objective of teaching (Barak and Shakhman, 2008).

This is not surprising because, although there is some agreement regarding the relevance of developing these skills in both teachers and students, thinking skills are not being sufficiently emphasised in teachers' preparation courses



(Barak and Dori, 2009). Therefore, if teachers are not promoting or improving their students' thinking abilities, it might be because they do not know how to do so or are unaware of the importance thereof. For this reason, this study aims to develop a Cognitive Acceleration course within the context of initial teacher training in order to explore the learning experiences of prospective primary teachers during the course.

The study was carried out in Chile, where teacher training has been widely characterised as being poor and weak, not only by national scholars (Avalos, 2003, Contreras et al., 2008, Ortúzar et al., 2009, Bellei and Valenzuela, 2010, Montecinos et al., 2010, Peirano, 2010, Cabezas and Claro, 2011), but also by international organisations (IEA, 2003). This topic will be further addressed in the next chapter.

## **Research questions**

My study looks at the development and trial of a thinking skills course for pre-service primary teachers. The main research question that this thesis addresses is: How do prospective teachers perceive a cognitive acceleration approach in relation to teaching and learning?

This main research question could be divided into the following specific research questions:

1. How do the formal reasoning skills of prospective teachers change after experiencing a course on cognitive acceleration activities?
2. How do the views of prospective teachers towards teaching and learning change following a cognitive acceleration intervention programme?
3. How do the attitudes of prospective teachers towards the teaching of mathematics develop following an intervention that uses a cognitive acceleration approach?
4. How do the attitudes of prospective teachers towards developing thinking change following an intervention that uses a cognitive acceleration approach?

## Research objectives

In order to explore those research questions and to structure my study, there were six main research aims:

- i. Firstly, I adapted the Cognitive Acceleration in Maths (CAME) programme for working with prospective primary teachers in Chile. CAME activities were originally designed for being used with school students in the UK. Therefore, they not only had to be translated into Spanish, but also needed to be adapted for working with older students in a completely different context. One of the major features of each CAME activity is that it mainly engages students by introducing a story or an imaginary context that presents the activity as a real problem and not simply as a mathematical one. This “hook” cannot be lost and, for that reason, I changed some aspects of each activity in order to present them as real problems that students might face in their future careers as teachers.
- ii. The second objective was to explore any changes in the prospective teachers’ formal reasoning skills. Cognitive acceleration programmes are generally described as interventions that affect the students’ general thinking ability.
- iii. The third objective was to explore how prospective teachers view a cognitive acceleration approach in terms of experiencing the activities as learners. This aim looked at the experience that prospective teachers had as learners during the CAME course.
- iv. The fourth objective was to explore how prospective teachers viewed a cognitive acceleration approach in terms of the application of the activities to the teaching and learning of mathematics. In contrast to the previous objective, this one examined their experience of the course in relation to their training as teachers, in the sense of attempting to discover how plausible and useful they think the approach is for their future classrooms and students.
- v. The fifth objective was to explore how prospective teachers view a cognitive acceleration approach in terms of their confidence in

teaching mathematics. This aim attempted to discover out if prospective teachers felt there was any change in their ability to teach mathematics after participating in the course.

- vi. Finally, the study explored how prospective teachers' views regarding the importance of developing thinking changed following a cognitive acceleration course. This aim is related to the previous one, in the sense that I attempted to investigate whether teachers recognised any changes in terms of their views on promoting thinking in their classrooms after participating in the CAME course.

### **The structure of this thesis**

This thesis is organised into seven main chapters after the introduction of the topic and the research question this study explores. The first chapter describes the context where the study was developed, in order to argue that using the CA approach could be beneficial for training prospective teachers, considering the evidence available regarding the current educational situation in Chile.

The next two chapters are devoted to the literature review which, in general terms, has the purpose of placing the research questions in context. The literature review will also inform and support the planning, implementation and analysis stages of my study. Therefore, in those chapters, I will address the main topics that are related to this study. I will start by describing the nature of formal reasoning skills and how Cognitive Acceleration programmes have tried to encourage the development of these types of skills in their participating students. I will then discuss the skills and the knowledge that a teacher should have in order to effectively promote meaningful learning inside the classroom.

The literature review will be followed by the methodology chapter. In this chapter, I will make explicit my assumptions as a researcher and the paradigm from which I will approach this study. I will then move to the description of the methods chosen for this research and the research design. I will then describe my participants and the selection process I used to recruit them. By the end of the chapter, I will explain the ethical procedures that guided the conduction of the

entire research process and how I analysed my data in order to secure validity and reliability.

The fifth chapter will be devoted to the results of this study and is intended of showing the reader how the methods described in the preceding chapter led me to arrive at the results I present. The first part of the chapter will present the main findings from the participants I interviewed, the learning journals they wrote during the CAME sessions and the field notes I wrote during the course. Secondly, I will present the statistical analysis I conducted, based on the Science Reasoning Task results for the experimental and the comparison group at the beginning and at the end of the course. Finally, I will use the case of one of the prospective teachers, Sarah, as an example of how it was possible to observe changes in terms of her views and attitudes about learning, teaching and thinking.

The results section will be followed by the discussion chapter, which will generate a deeper analysis of and reflection on the results described in the previous chapter. In this context, I will present the implications of my findings for the design and development of initial teacher training sessions that are oriented to primary mathematics, and to the promotion of thinking skills for prospective teachers. I will also discuss the relevance of my results within the context of Cognitive Acceleration research and how this research has contributed to it and to the educational field in general.

Apart from describing what this research study is adding to that which we already know in this field, it is always relevant to present the limitations of a study and to explore the scope of its conclusions. For this reason, the seventh chapter will not only deal with this topic, but will also describe what I learnt when conducting this research. The chapter will conclude with suggestions for various new lines of research that this study did not include and which would be interesting to explore in the light of the findings discussed earlier.

Finally, the conclusion chapter is a short but informative summary of the entire thesis.

## **II. Initial teacher training: Why could CAME be useful?**

This chapter tries to answer the question of how using the CAME approach could be beneficial for training prospective teachers in general and in Chile in particular. In answering this question, I will use three main arguments. The first is that teachers' development and change takes a lot of time and effort. Therefore, if educational systems want teachers to teach thinking skills in their classrooms, showing teachers how to do this in their initial training could be more effective and might have a bigger impact. The second argument is that cognitive development continues after adolescence, therefore using a cognitive acceleration approach with prospective teachers makes perfect sense from a neurological point of view. That claim will be supported by evidence from neuroscience that shows that, contrary to what has been believed in the past decade, new neuroimaging techniques have been showing that the brain is still plastic throughout adulthood. Finally, the last argument is related specifically to the current educational situation in Chile and the way in which teachers are being trained to qualify as primary teachers. In the context I will describe, I think that using the CAME approach could be a major contribution and improvement to the way in which teachers are being trained in Chile.

### **Teacher development and change**

Nowadays, it is widely recognised that promoting thinking skills is one of the main goals of effective teaching for the 21<sup>st</sup> century (Crump et al., 1988, Fennema et al., 1996, Franke et al., 2001, McGuinness, 1999). Thus, many schools have implemented programmes with different levels of structure that aim to develop these kinds of abilities in their students (Crump et al., 1988, Adey and Shayer, 1990, Fennema et al., 1996, Adhami et al., 1997, Zohar, 1999, Ferretti et al., 2001, McGuinness, 1999).

Although several of these programmes have produced positive results, one of the biggest challenges they have faced is training teachers to change their classroom practices. Zohar (1999) claimed that the successful implementation of

thinking skills programmes has been affected by the difficulties in developing the necessary teaching skills through in-service and pre-service instances. According to McGuinness (1999), part of these difficulties can be explained by the fact that teachers' previous knowledge is often put into question when they are trying to adopt and promote a more constructivist learning environment in their classrooms, which is an indispensable condition for many thinking skills initiatives.

An illustration of these difficulties is reported by Crump et al. (1988) in a US school district. The researchers ran a programme that intended to foster high order thinking skills in their secondary school students. In order to do this, their teachers participated in a training programme that lasted 18 hours, in which they covered 19 different skills (decision making, planning, productive thinking and so on) through direct instruction. The research team found that, even when teachers demonstrated profound levels of understanding regarding the method and its purposes, they experienced high levels of complexity when trying to apply the method or teaching model in their classrooms.

These findings are not surprising, given the amount of literature that has described the difficulty of accomplishing effective and long-lasting teacher development and change (Guskey, 2002, Hargreaves, 2004, Fullan, 2007, Opfer and Pedder, 2011). However, if adequate conditions are assured, it is possible to promote teachers' learning and their professional growth. In this context, several researchers (Stein and Wang, 1988, Clarke and Hollingsworth, 2002, Guskey, 2002) have proposed elements and models that try to explain and make explicit the combination of factors that play a role in the complex process of teacher change.

Stein and Wang (1988) claim that a precondition for encouraging successful teacher change is the belief that they are able to change if they receive adequate training and support. If these two elements are secured, the next step is to develop the teachers' commitment or motivation to not only adopt the intended skills, but to also put them into practice in their classrooms. In this process, teachers' perceptions regarding their efficacy in implementing new practices and the value of such new practices are crucial. In other words, if teachers do not feel confident about their ability to translate what they have learned to their

classrooms, or do not see that learning as being valuable, it is not likely that they will actually use it.

Short-term training is another factor that has been consistently identified as hindering teachers' development (Stein and Wang, 1988, Richardson, 1998). With regard to this point, Opfer and Pedder (2011, p. 384) argue that "most research has concluded that activities that effectively support teachers' professional learning need to be sustained and intensive rather than brief and sporadic". If principals, policy-makers or other educational authorities want a programme to succeed and to become part of the institution, they need to provide not only long-term training, but also the necessary support and a well designed and viable follow up structure (Clarke and Hollingsworth, 2002). Teachers are not individual and lonely agents; rather, they are part of a learning community that should encourage its members to support each other in order to enhance their students' learning.

Opfer and Pedder (2011) also pointed out that providing teachers with field and classrooms experiences relevant to their practice is another requisite for changing their practice. In this sense, training experiences that take teachers out of their classrooms and to training institutions where they hear someone else speaking about how to do something new are unlikely to succeed. Teachers need to experiment with the new practices that are being promoted by themselves and inside their own classrooms, otherwise the new knowledge will be only theoretical and will not be transferred to the classroom. In addition, teachers need the time and the opportunity to reflect on their current practice, because if they do not feel the need to change or do not think that improvement could be made, they are not likely to be inclined to change their habits. The final important point is that all change, or intention to change, should take place in a safe and protected environment in which the practice and not the person is being questioned. Only in this context will teachers be unthreatened by change.

All of these factors clearly point out that promoting teachers' development and change is an extremely complex enterprise that requires a lot of effort. This, in conjunction with the evidence that shows that teachers tend to reproduce the teaching style they have experienced as students (Hiebert et al., 1996, Remillard, 2000, Goulding, 2002, Collopy, 2003, Burgess and Mayes, 2008, Henderson and Rodrigues, 2008, Opfer and Pedder, 2011), makes it reasonable to claim that a good place to start training teachers to develop thinking skills in their students might be during their pre-service education as prospective teachers. This would give them the opportunity not only to experience a classroom environment as students and a teaching style that encourages thinking, but also to develop the necessary skills for promoting thinking in their students in their classrooms in the future.

### **Cognitive development continues after adolescence: evidence from neuroscience**

A decade ago neuroimaging techniques were not as developed as they are today and, for that reason, most evidence about the development of the brain was collected post mortem (Blakemore and Choudhury, 2006). In that context, given that during childhood the brain was completely plastic, most neural connections were just developing, and that the volume of the brain did not increase substantially after 5 or 6 years old (Casey et al., 2000, Yurgelun-Todd, 2007), it was traditionally believed that most part of cognitive development occurred during those years. As a consequence, those first years were always considered as the most critical period for cognitive development to occur.

However, more recent evidence provided by new neuroimaging techniques have shown that adolescence is also a critical period in terms of the potential for cognitive development (Choudhury et al., 2006, Paus, 2005). In fact, according to Yurgelun-Todd (2007) "The adolescent years are characterized by the maturation of emotional and cognitive abilities that provide the developing individual with capacities needed for independent functioning during adulthood (...) Adolescence is a critical period for maturation of brain processes that underlie higher cognitive functions and social and emotional behavior" (p. 251).



The only change in terms of brain volume that occurs during the second decade of life and is related to complex cognitive processes is the dorsolateral prefrontal cortex (Casey et al., 2000, Yurgelun-Todd, 2007). However, even when it is true that adolescence and adult cognition is partly explained by the development and growth of the prefrontal cortex that is tightly associated to higher cognitive processes, it is also now understood that all those higher functions are supported by a more integrated network of neural connections that involve a variety of brain regions and not the prefrontal cortex exclusively (Luna et al., 2001).

In this sense, the application of magnetic resonance imaging techniques has made possible to claim that more complex cognitive abilities observed from adolescence onwards are mostly explained by the development of executive functions that include a variety of essential abilities such as organization, planning, hypothetical thinking, metacognition, abstract thought, deductive reasoning, cognitive flexibility, decision making, information processing, etc. (Choudhury et al., 2006, Durston and Casey, 2006, Luna et al., 2001, Steinberg, 2005, Yurgelun-Todd, 2007). Those improvements in the executive function are therefore supported by new neural connections in the pre-frontal and frontal cortex and not only by a higher brain volume as a whole. In fact, that higher amount of connections has been observed in an increase of white matter that reveals axons myelination and as a result produces faster information processing (Steinberg, 2005).

The fact that white matter volume increases constantly until the first years of adulthood, has been considered by scientists as the evidence that supports the idea that cognitive development is an ongoing process that lasts much longer than what has been thought (Casey et al., 2005). According to Choudhury et al. (2006) “These changes presumably reflect ongoing myelination of axons by oligodendrocytes enhancing neuronal conduction and communication (...) Connections are being fine-tuned with the elimination of an overabundance of synapses and strengthening of relevant connections with development and experience” (p. 106).

As Choudhury et al. (2006) states, if experience can promote new neural connections and strengthen old ones making them faster, using a cognitive

acceleration approach with prospective teachers is a plausible tool not only for modelling social-constructivist teaching practices but also for promoting cognitive development. In this sense, what I am trying to claim is that evidence from neuroscience and neuroimaging show that it is not too late for prospective teachers to improve their cognitive processes.

## **The context of the study: Chile**

### *A. Description*

In Chile, higher education is provided by three different types of institutions:

- i. Universities that are authorised to grant Bachelor's (8-10 semesters), Master's (2-4 semesters) and Doctoral degrees (8-10 semesters)
- ii. Professional Institutes that teach professional courses that usually last about 8 semesters but which are not entitled to grant Bachelor's degrees and issue professional diplomas instead
- iii. Technical Centres, which develop technical programmes that are not usually longer than 4 semesters.

In this context, it is possible to find a wide range of different programmes that provide initial teacher training. In fact, in 2008, there were 698 different teacher education programmes (pre-school, primary, secondary, special education and so on) offered by 95 institutions (Peirano, 2010).

Of the total amount of teacher training programmes, 198 correspond to primary education and are offered by 5 professional institutes and 47 universities (Varas et al., 2008). Given the variety of programmes, it is possible to find many different structures and curricula for training prospective teachers. For example, according to Telias and Valenzuela (2008), 34.5% of current schoolteachers studied a programme that lasted for 6 semesters or less, 43% studied for between 7 and 8 semesters and only 21.5% studied for 9 semesters or more. In addition, 63% of current teachers studied a programme that required physical attendance and which was attended during the day. In this sense, 37% of the teachers

obtained their qualifications at semi-present or full-distance programmes, which do not demand any attendance at all.

Notwithstanding the aforementioned, I will describe a traditional primary teacher-training programme offered by one of the best Chilean Universities by way of illustration. Usually, Bachelor's degrees in primary education last for 8 semesters and students are certified as general teachers on completion of the course, which means that they are not specialists in any particular subject. In addition, they can study for an additional year (2 semesters) in order to attain a specialisation. This qualification authorises them to work as schoolteachers with students from year 3 to year 10.

Each semester is composed of different courses that can be classified into four broad knowledge categories, namely general, professional, specific and practical. General knowledge is usually understood as the comprehension of the social, historical, ethical and political bases of education and of the teaching profession. In turn, professional training is the development of knowledge related to pupils, curricula and evaluation models. Specific knowledge refers to the content and pedagogical knowledge that a teacher should develop regarding the subject(s) s/he teaches. Finally, practical knowledge is the understanding developed during the teaching activities in which the student was involved at a school and reflection regarding such teaching practice (Contreras et al., 2008).

Usually, the practical training (internship or apprenticeship) is gradually introduced during the programme. For example, during the first year, student teachers have theoretical courses only. Then, in the second year, they are introduced into a school classroom as an observer. In the third year, they are teachers' assistants and, finally, in the fourth year they take on the responsibility of being in charge of a class. It is important to note that these practical experiences usually do not represent more than 20% of the time that they spend at the university attending courses (and excluding homework). This proportion is true only in the case of traditional programmes, as Ortúzar (2009) states that 63% of the teachers who have less than 12 years of teaching experience studied a teacher education programme in which practical courses constituted less than 10% (for more details see Table 1).

**Table 1: Example of Initial Teacher Training Programme Structure**

Semester		Courses			
<b>I</b>	Education Theory	Education, Culture, and Society	Psychopedagogy of development	Basic Mathematics	Visual Artistic Language
<b>II</b>	Psychopedagogy of learning	Education and Philosophy	Curriculum	Childhood, Language and Literature	Internship I: Vocation and Teaching Profession
<b>III</b>	Educational Evaluation	Teaching Theories	Musical Language	Natural Sciences	Elective: Other discipline
<b>IV</b>	Pedagogy for diversity	Mathematics Didactics I	Artistic Education Didactics I	Internship II: Educative Institution Experience	Theology
<b>V</b>	Educative Orientation	Language Didactics I	Social Sciences	Elective: Other discipline	Elective: Other discipline
<b>VI</b>	Natural Sciences Didactics	Social Sciences Didactics I	Technology Education	Internship III: Didactics Experience	Elective: Other discipline
<b>VII</b>	Education throughout history	Politics and Educative Organisations	Physical Education Didactics I	Technology Education Didactics I	Elective: Other discipline
<b>VIII</b>	Professional Ethics	Research Seminar	Internship IV: Pedagogic Internship	Elective: Other discipline	

(Pontificia Universidad Católica de Chile)

With regard to the demographic characteristics of prospective Chilean teachers, they are predominantly female, particularly in the case of special education and pre-school teachers. The exception to this trend is the case of secondary teachers, where the gender distribution is similar with approximately

50% of the students being male (see Table 2). In terms of age, student teachers are, on average, 25 years old.

**Table 2: Number of students enrolled in Bachelors' of Education degrees in 2009**

	Female	Male	Total	% Female
<b>Primary Education</b>	16,641	4,758	21,399	78
<b>Special Education</b>	6,158	294	6,452	95.4
<b>Secondary Education</b>	26,323	25,567	51,890	51
<b>Pre-School Education</b>	9,296	32	9,328	99.7

Higher Education Information System (SIES), Ministry of Education (2009).

### *B. Teacher education programme quality*

It is widely recognised by educational leaders that Chilean student teachers are not being sufficiently prepared to promote their pupils' learning (Avalos, 2003, Contreras et al., 2008, Ortúzar et al., 2009, Bellei and Valenzuela, 2010, Montecinos et al., 2010, Peirano, 2010, Cabezas and Claro, 2011). One of the most important factors that might be considered to be contributing to this situation is the lack of regulation of initial teacher training programmes. Although each programme should be certified by the National Commission of Accreditation (CNA), this process only verifies that Universities and Professional Institutes are actually achieving their self-set goals. In this sense, there are no standards and/or benchmarks against which the current programmes could be compared.

The variety of entry requirements for each teacher education programme could be considered as a measure of the lack of regulation. In Chile, higher education institutions that are mainly funded by public resources ask their applicants for a minimum score of 450 points in the University Selection Test (PSU). PSU is a national examination that assesses school content knowledge through four different tests, namely language, mathematics, natural science and social science. The first two tests are compulsory and the second ones are optional, depending on the programme to which the student is applying. The total

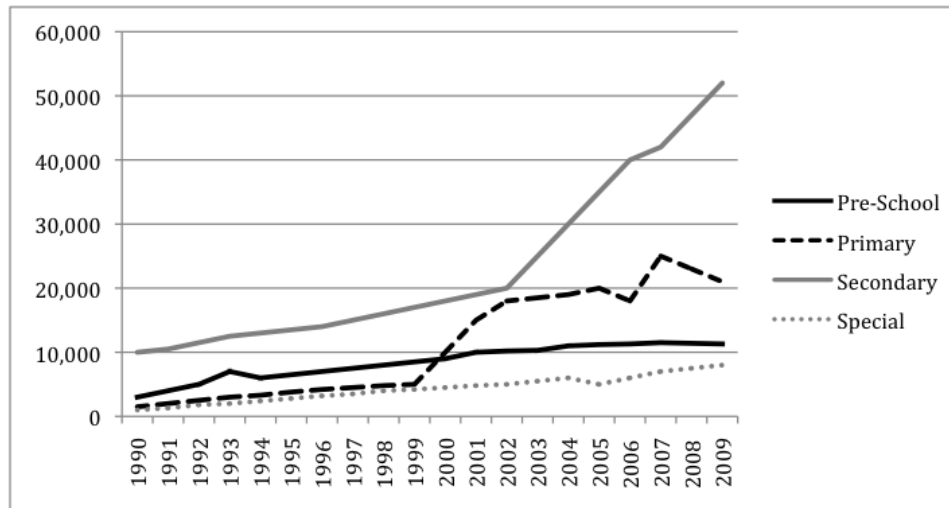
score for applying to a University programme is calculated based on the score in each test and the weight thereof in the total score. For example, the same programme that I used for illustrating the structure of a teacher-education training programme assigns 20% to high-school grades, 30% to the language test, 30% to the mathematics test and 20% to the natural or social science test.

However, higher education institutions that are only partly funded by public resources or that do not receive any public funding are not required to ask for PSU scores. In other words, if private higher education institutions are mainly funded by the fees they charge to their students, they have a strong reason to fill all available vacancies, even if the applicants do not meet all the requirements. Otherwise, their financial solvency might be compromised. As a result, there are more vacancies for initial teacher training programmes in the educational market than there are qualified applicants. For example, in 2007, only 53.7% of teacher education programmes demanded that their applicants take the University Selection Test (Bellei and Valenzuela, 2010).

Thus, poor regulation and the absence of exigent entry requirements have become a major problem, particularly in the last 30 years, since there has been an explosion in the number of teacher education programmes offered and the number of students enrolled in them (see Figure 1). This explosion has occurred in all types of teacher education programmes (pre-school, primary and secondary). In this sense, the number of students who were enrolled in a primary education programme between 1997 and 2007 increased from 4,952 to 13,299. In addition, during the same period, the students who were registered for secondary education programmes increased from 14,210 to 51,301 (see Figure 1).

Until 1980, Chile had only 14 universities that were directly funded and regulated by the State; today, these are called ‘traditional’ universities. However, in 1980, a law was passed that established the requirements for and authorisation to open private universities, which evidently facilitated the appearance of new universities in response to an increasing demand for tertiary education. As a result, there were 225 teacher education programmes (pre-school, primary, secondary, special education) in 1999 but, in 2008, there were 698 programmes offered by 95 different institutions (Bellei and Valenzuela, 2010).

**Figure 1: Enrolment Evolution in Initial Teacher Education Programmes**

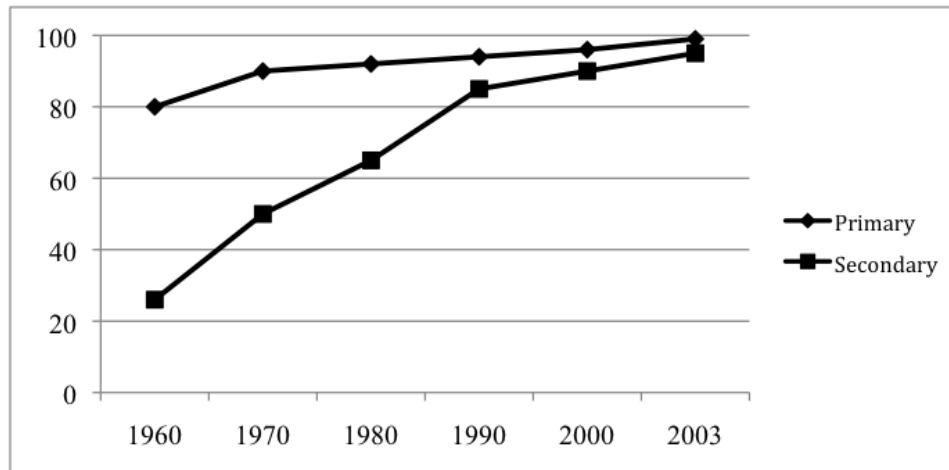


Higher Education Information System (SIES), Ministry of Education (2009).

This explosion in the offer of teacher training programmes was prompted by the increasing trend in school enrolment, which was promoted by law and which established there should be 8 years of compulsory education (primary and secondary) instead of 4 (Brunner et al., 2006) (see Figure 2). In this context, the growing school system needed many new teachers and the demand was satisfied by the existent facilities for opening and running new teacher education programmes. However, as I have pointed out, this explosion has not been accompanied by strong supervision and regulation initiatives that can ensure quality.

At the same time, over the last thirty years, most Chilean primary teachers (who teach students from 6 to 12 years old) have been trained as general teachers, which means that they are not specialists in any particular subject. In contrast, secondary teachers (who teach students from 13 to 18 years old) need to be experts in a specific subject. As a result, primary teachers are prepared during four years to teach ten different subjects to students who are in seven different grades, while secondary teachers are trained over five years to teach only one subject to students who are in five different grades (Cox, 2007).

**Figure 2: School enrolment as a percentage of the corresponding age group**



(Brunner et al., 2006, p. 27).

The research literature has broadly suggested that teachers' content and pedagogical knowledge is an essential component of effective teaching practice (Cox, 2007, Darling-Hammond and Bransford, 2005). In fact, all the school systems that obtained the best results in educational tests, such as TIMSS and PISA, prepare their primary teachers well in terms of disciplinary knowledge (OECD, 2005). Given the relevance that teachers' subject content knowledge has for their pupils' learning, it is not surprising that the lack of disciplinary expertise of Chilean primary teachers has been mentioned by the Organisation for Economic Co-operation and Development as one of the three most important problems in Chilean education (OECD, 2004).

Recently, the Chilean Government has been trying to implement various actions in order to improve the quality of future teachers. In this context, efforts aimed at evaluating teachers' performance have taken a central place in educational policies. The last programme, which is still in the experimental phase, is called *Inicia* and aims to evaluate future teachers' content and pedagogical knowledge when they are in the last year of their Bachelor of Education, or are in the process of receiving a diploma. This test was implemented for the first time in 2008 and the results showed that prospective teachers were able to respond correctly only to 50% of the questions (Peirano, 2010, p.72).



Table 3 and Figure 3 show that Inicia results have a positive and strong correlation with PSU scores. In this sense, the students who obtain low scores on the Inicia test are usually the same students who get low PSU scores. This might imply that initial teacher training programmes are not developing the necessary skills of prospective teachers during their Bachelors of Education programmes, because the best predictor of their performance in the qualification test (Inicia) is the result they attained in a test they took before entering the teacher training programme (PSU).

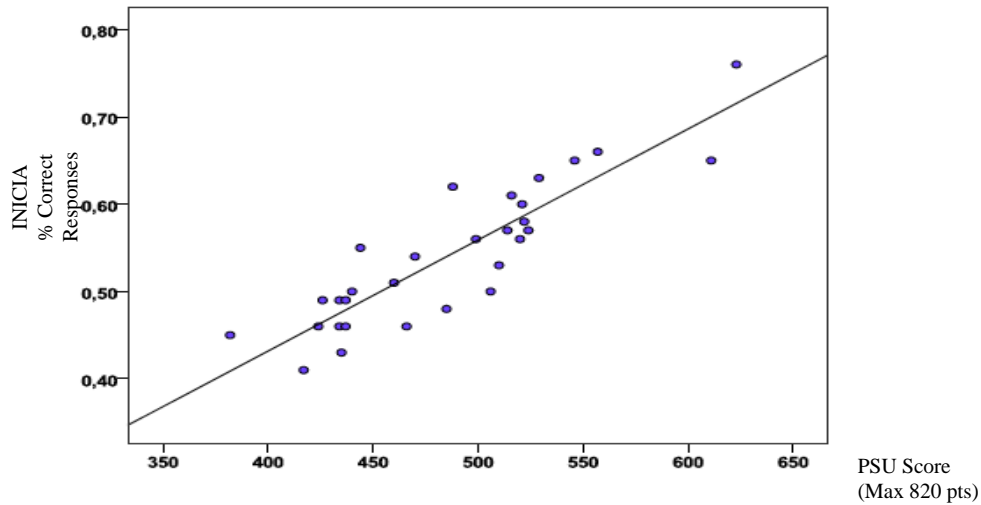
According to these results, the students who obtained the lowest scores in the University Selection Test (PSU) at the beginning of their higher education are the same students who obtained the lowest scores in the Inicia test at the end of their teacher training programmes. Based on this data, it could be suggested that initial teacher training programmes are not succeeding in the aim of bridging the gap between students with different entry performance levels (PSU scores), since the final results (Inicia scores) are largely explained by prior achievement. These results are extremely distressing, particularly when taking into consideration research evidence that has concluded that one of the teachers' characteristics that helps to explain the variation in students' achievement is their performance as school students and their results in University admission tests (Goldhaber, 2008). This could mean that prospective teachers who obtained low results in their University admission tests (in this case, PSU), might have a negative impact on their future students' achievement.

**Table 3: Prospective teachers' scores in PSU and INICIA tests**

PSU Score	PSU	INICIA
	N° Correct Responses	% Correct Responses
<b>&lt; 450</b>	16.7 – 26.7	40%
<b>451 - 500</b>	26.7 – 41.3	45%
<b>501 – 550</b>	41.3 - 58	51%
<b>551 &lt;</b>	58 -	63%

INICIA 2009, Ministry of Education, Chilean Government.

**Figure 3: Correlation between INCIA and PSU scores**



INICIA 2009, Ministry of Education, Chilean Government.

These results are consistent with those obtained by prospective Chilean mathematics teachers who participated in the recent Teacher Study in Mathematics (TEDS-M) carried out by the International Association for the Evaluation of Educational Achievement (IEA). The purpose of the study was to compare teacher education in the participating countries, with a special emphasis on the preparation of teachers of mathematics at the primary and lower secondary levels. Consequently, TEDS-M evaluated future teachers' content and pedagogical knowledge of mathematics when they were almost ready to receive their diplomas. Chilean primary teachers were ranked 15th out of the 16 participating countries and lower secondary teachers occupied the last position (Babcock et al., 2010).

### *C. Initial teacher education applicants*

The poor quality of the programmes is particularly serious if one takes into consideration that, in Chile, the teaching profession currently has extremely poor social standing (Avalos and Assael, 2006, Bellei and Valenzuela, 2010, Cabezas and Claro, 2011, Peirano, 2010). This situation can be understood in light of

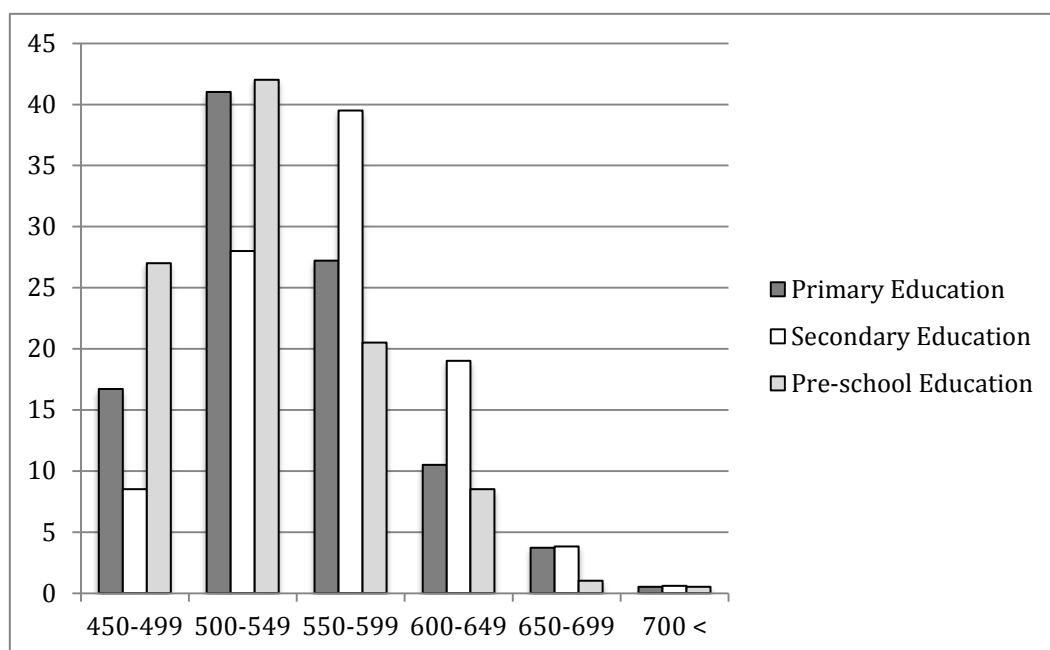
certain educational measures that were implemented during the military government between the years 1973 and 1990. Firstly, public schools were administrated directly by the Ministry of Education until 1973. Hence, teachers were considered by others and by themselves as public servants. For this reason, they enjoyed an important position in society.

In 1980, however, in order to decentralise the school system, schools were transferred to local authorities (Mayors) who had the authorisation to hire and fire teachers. Given the political and social situation in Chile at that time, dismissals were justified for reasons other than teacher performance. In addition to losing their social recognition as public servants when the schools were transferred to local authorities, teachers' salaries no longer corresponded to those of the scale of public functionaries' incomes. Thus, teachers' salaries in the 90s were significantly lower than they were in the early 70s (Avalos and Assael, 2006).

The last reform that negatively affected the reputation of the teaching profession was that Bachelor of Education programmes lost their university status and were considered to be part of tertiary education. In other words, all the Faculties of Education that existed at that time were converted into professional institutes. According to Avalos (2006, p. 256), "Although this decision was reversed towards the end of the military period, it had a dramatic effect on teachers' social status and on the quality of applicants to teaching that persisted way into the 1990s.

These factors, in conjunction with others like low salaries in comparison to other professionals and poor quality job conditions, have resulted in teacher training programmes not being able to attract talented applicants. This can be seen by looking at the average PSU scores that future teachers obtained when they applied to Bachelor of Education programmes (See Figure 4 and Table 4). In addition, only two institutions that offer teacher-training programmes have students with average PSU scores that are higher than 600 points (of a total of 820 points).

**Figure 4: Percentage of future teachers who obtained different ranges of PSU scores**



Higher Education Information System (SIES), Ministry of Education (2009)

**Table 4: PSU score meaning in terms of the number of correct responses**

PSU Score	PSU
	N° Correct Responses Out of 150
	(80 in Language and 70 in Mathematics)
<b>450 - 499</b>	16.7 – 26.7
<b>500 - 549</b>	26.7 – 41.3
<b>550 - 599</b>	41.3 – 58
<b>600 - 649</b>	58 – 73.3
<b>650 - 699</b>	73.3 – 84
<b>700 &lt;</b>	84 or more

Higher Education Information System (SIES), Ministry of Education (2009).

The evidence provided could be considered strong enough to suggest that initial teacher training programmes in Chile are not attracting the best applicants and are not sufficiently preparing student teachers to face the challenges they will encounter in their future practice as school teachers. However, prospective

teachers have been evaluated only through tests (PSU, Inicia and TEDS-M) that, most of the time, have limitations in terms of providing a completely accurate picture. In addition to the tests' limitations, literature on initial teacher training states that, even though both content and pedagogical knowledge are two important factors that contribute to effective teaching practice (Cox, 2007, Darling-Hammond and Bransford, 2005), they cannot be considered to be the only ones. Teaching practice is highly complex and, for this reason, a variety of factors can help to explain a teacher's effectiveness in promoting students' learning.

One of these factors is teachers' cognitive processes. According to Green (2006), another essential element of effective teaching is that teachers should rely on metacognitive strategies, since they should become reflectively and metacognitively involved with the content, so as to ask themselves professional questions regarding the cognitive and pedagogical processes. Related to this, teachers who practice a strongly metacognitive discourse promote the formation of students who are much more capable of understanding their own and others' opinions (Olson and Astington, 1993). In this context, the possibility of exploring and developing prospective teachers' thinking skills gains relevance, since these are abilities that teachers will need and should use during their professional experience in order to develop their pupils' thinking and learning abilities.

Professional development programmes that Cognitive Acceleration interventions have implemented in order to train teachers to deliver them, have been successful in engaging in-service teachers in the kind of cognitive process and socioconstructivist teaching models they are then supposed to promote in their own classrooms. For this reason, implementing the approach with prospective teachers is fundamented by the idea of preparing future teachers for promoting not only these skills in their students but also for evidencing classroom practices that are coherent with socioconstructivists perspectives.

### III. Literature Review: Part I

This research describes the experience of using an approach that tries to promote the development of formal thinking skills in the participating students. In this context, it is important to start by describing formal reasoning skills and what has been said in the literature regarding the degree to which they are usually promoted. Even though most people would agree that developing thinking is an important goal for meeting the requirements of the present society, it still is quite a wide concept that needs to be narrowed down and delimited for the purposes of this research.

#### **Formal reasoning skills**

Jean Piaget and his colleagues were among the first to use the term ‘formal reasoning skills’ and to describe the processes whereby cognitive structures are developed (Anderson, 2003). For this reason, Piaget has been considered by others (Beilin, 1990, Shayer, 1993, Lourenco and Machado, 1996, DeVries, 2000, Shayer, 2003, Dawson-Tunik et al., 2004) as one of the most important contributors to the progress of developmental psychology. Furthermore, Beilin stated that “assessing the impact of Piaget on developmental psychology is like assessing the impact of Shakespeare on English Literature” (1992, p.191).

This work is mainly based on the Piagetian concept of formal thinking skills, which is why Piaget’s theory and epistemology will be described in this section.

#### *A. Jean Piaget and his epistemological theory*

As a result of great and long effort, Piaget developed a theory that describes the progress of cognitive development from birth (Anderson, 2003). Based on his findings, he stated, “Intellectual structures between birth and the period of 12-15 years grow slowly, but according to stages in development. The

order of succession of these stages has been shown to be extremely regular and comparable to the stages of an embryogenesis” (Piaget, 1972, p.41). In other words, Piaget (1972) identified the sequence of stages through which intellectual structures advance in every child. However, that does not mean that each person’s structures move from one stage to the next at the same time or at the same age. While the sequence is predetermined, the speed of each person’s progress is influenced by an immense variety of factors that make each individual’s progress unique.

The first stage described by Piaget was the sensorimotor stage, which is present in children before the appearance of language (from birth to two years of age). As language and thus representational ability have not yet developed, the only things that exist in the intelligence of the child are actions and the coordination among them (Anderson, 2003). In this sense, at this time the only sources children have for understanding their surrounding world are sensory perceptions and motor activities. Thus, Piaget (1972) characterised this stage’s behaviours as “instrumental”, because children react to their sensory perceptions via motor activities.

With the development of language, the child also acquires the capacity for representational thought and symbolic play. This milestone marks the beginning of a new stage, called the pre-operational stage (2-7 years old). Behaviours are no longer simply actions that are present in the physical world, as children are now capable of interiorising actions and of having a representation of them in their minds. However, the operations at this stage are still not entirely complete and, because of this, Piaget (1972) called this stage pre-operational. When we talk about operations, we mean actions that are reversible, such as adding and subtracting. The lack of reversible operations implies conservation, which means that, at this stage, children believe that if the shape of an object changes, the weight and the quantity of matter will also change. In addition, children are not able to comprehend the concept of transitivity; for example, the ability to conclude that  $A \leq C$  if they know that  $A \leq B$  and  $B \leq C$  (Piaget, 1972).

As language is the starting point of the pre-operational stage, the logic of reversible actions marks the beginning of the concrete operations stage (7-11

years old). The former is characterised by the establishment of new, coherent and stable structures that allow children to execute a wide range of operations such as classification, ordering, construction of natural numbers, certain types of causality and so on. These operations have various degrees of reversibility, such as inversion, negation and reciprocity. The result of the former is an annulment; for example,  $-X + X = 0$ . In turn, the latter characterises operations of relations, such as the case of if  $A = B$ , then  $B = A$ . It is important to note that, although children are now capable of operations, they are still concrete in terms of their thinking processes. This means that children's reasoning patterns are characterised by operating upon the basis of objects (relations, classes and numbers) and not by thinking about them hypothetically without the necessity of knowing if they are true or false. Another particularity of this stage is that the child is able only to relate different objects or different elements of objects when they are immediately adjacent. In other words, although these concrete operations imply sorting and establishing relations, this is merely between close items and not to other ones within a given range (Piaget, 1972).

The last stage described by Piaget is the formal operational stage, which is reached during adolescence at about 14-15 years old. He stated "The principle novelty of this period is the capacity to reason in terms of verbally stated hypotheses and no longer merely in terms of concrete objects and their manipulation" (Piaget, 1972, p.42). This change is a very important one, because the universe of reasoning becomes independent of the real world. The adolescent is now able to think in terms of what could be possible and not only about what it is real which, in turn, makes him or her capable of anticipating the consequences of a hypothetical premise without necessarily judging the truthfulness or the falseness of it. All these changes produce qualitatively relevant progress in the social sphere. Hypothetical reasoning transforms other's points of view into arguments that can be understood and evaluated in terms of the consequences that can logically be deduced from them. This does not necessarily mean that adolescents have to share others' opinions, but they can now think about and discuss them with others.

As an illustration of the different stages and the particular type of thinking described above, I will present one of the experiments developed by Inhelder and



Piaget and the responses that children at different stages gave to it. Inhelder and Piaget (1958) developed an experiment called “The law of floating bodies and the elimination of contradictions” with the purpose of studying children’s understanding of the law of floating bodies (Anderson, 2003, pp. 20-45).

*Example: The law of floating bodies and the elimination of contradictions*

The law of floating bodies and the elimination of contradictions states that objects float if their specific gravity (the relation between the weight of the object and an equivalent volume of water) or density (the relation of weight to volume) is less than that of water. Therefore, the law expresses a relationship between two classes: the class of bodies whose density is greater than the density of water and the class of bodies whose density is less (Bergling, 1998b).

Piaget and Inhelder (1958) developed two types of experimental conditions depending on the age of the children who were being evaluated. The first was for younger children and consisted of showing them different kinds of objects, asking them to classify each object in terms of whether or not it floats in water. After giving an answer, the children were asked to justify their choices. Subsequently, the experimenters gave the children a container of water and encouraged them to perform their own experiments. At the end of the experiment, the children were asked to summarise their conclusions. The second experimental condition, which was used with older children, consisted of presenting them with objects to be classified as floating or not floating (Piaget and Inhelder, 1958).

Preoperational children tried to classify the objects in terms of floating or not floating, but were unable to do this successfully because they do not have the ability to understand class inclusion. Class inclusion is characterised by the capacity to think about the whole and its parts simultaneously; however, but preoperational children have not yet developed the conservation of whole. In addition, in order to be able to generate a set of inclusions, the child has to understand how similarities and dissimilarities determine the intensive properties of a class. Accordingly, classification at preoperational level is only preliminary.

As a result, the child uses subclasses without a hierarchy. For example, the child can give reasons for objects to float because it is their nature is to float (“Why does it float?: because it is a boat”) or because they are small, or light, flat, thin and so on. In contrast, s/he could also justify the object’s ability to float according to its weight, such as explaining that a key cannot float because it is too light. As can be noted, preoperational subjects often utilise contradictory explanations (Bergling, 1998b).

A child at the concrete level is not able to relate volume to weight in understanding the volume of the object equal to the volume of the water, because s/he has not yet developed the concept of the conservation of volume. Consequently, the child associates the weight of the object with the total volume of water, instead of relating it to that of the displaced water. This leads to explanations such as a boat that can float on a lake being too heavy to float on a river. This is related to the appearance of a new kind of contradiction, because the child will sometimes argue that the weight of the water will carry the object, while at other times, s/he will argue that the weight of the water keeps the object at the bottom. The impossibility of distinguishing between the concept of absolute weight and of specific gravity produces contradictions that explicate arguments like “metal always sinks”. When the subject is approaching the late concrete level, there is a tendency to try to find a unique explanation. This means that the child believes that objects with higher specific gravity are more “full” or “filled” than others. The former phenomenon is a consequence of the subject’s endeavour to relate the weight, which s/he has conserved, to the volume, which s/he has not conserved (Bergling, 1998b).

In contrast to the concrete level, the formal reasoning person is now able to relate the weight and volume of the object to the weight and volume of the water underneath the object, because s/he has already achieved the concept of formal conservation of volume that allows him or her to understand the concept of density. At this level, the subject has the capacity to take into account all the possible combinations and, through this process, to find the correct causal factor. This is because of the understanding that, if the volume of the immersed object is increased and its weight is unchanged, this will give the same result as increasing the weight while leaving the volume unchanged (Bergling, 1998b).

Based on the example of the law of floating bodies and the elimination of contradictions, it is possible to summarise, in very simple terms, that what Piaget did was to describe the kind of mental operations that children are able to perform at the different developmental levels he identified. Reaching the last stage - formal operations - is the desirable outcome for every person; therefore, the next section describes these formal operations and explains why they are important.

### *The 10 formal operational schemata and their relevance*

According to Piaget, there are ten qualitatively different formal operations: combinatorial thinking, control of variables, exclusion of irrelevant variables, coordination of frames of reference, notions of probability, notions of correlation, multiplicative compensation, equilibrium of physics systems involving three or more variables, proportional thinking and physical conservation involving models (Piaget and Inhelder, 1958).

In order to have a better idea of what Piaget meant by these reasoning patterns, I will briefly present the explanation given by Adey and Shayer (1994, pp.17-24) for each one. This is because I find their interpretation of the reasoning patterns to be simpler and clearer, and because I intend to use their intervention approach within my empirical study; thus, these explanations fit well with my intended approach.

- i. *Combinatorial thinking*: This reasoning pattern refers to the ability to select a combination of variables (permutation). For example, solving the following problem involves such ability: “A dog has different colored puppies born on the same day. They were born in the following order: Spotted (S), White (W), Tan (T) and Black (B). The puppies could have been born in any order. Write all of the possible ways in which the puppies could have been born” (Chiappetta and Russell, 1982, p.88).

- ii. *Control of variables*: Almost all experiments deal, to some extent, with variables that need to be controlled. For example, in Piaget's pendulum experiment, the person needs to evaluate the role that a group of variables (length, weight, push) plays in the pendulum's swing. In order to solve this problem successfully, the person needs to be able to change one variable at a time in order to check how the swing rate is affected.
- iii. *Exclusion of variables*: The ability to control variables is usually associated with being able to identify that certain variables do not have any effect on the problem being observed.
- iv. *Frames of reference*: This refers to the ability to rely on memory when facing a new situation. By so doing, we are placing that new situation inside a specific "frame" that helps us to understand it, to make deductions and to react if our expectations are not met.
- v. *Probability*: In the case of probability, formal thinking is related more to the ability to understand the probabilistic nature of an event than it is to a particular numerical answer.
- vi. *Correlation*: This is the ratio of confirming and disconfirming cases that describe the relationship between two variables. For example, the study of a new treatment that aims to prevent gestational diabetes will have four different groups, as follows:
  - (a) Women who have not participated in the treatment and who have gestational diabetes (confirming cases)
  - (b) Women who have not participated in the treatment and who do not have gestational diabetes (disconfirming cases)
  - (c) Women who participated in the treatment and who have gestational diabetes (disconfirming cases)

- (d) Women who participated in the treatment and who do not have gestational diabetes (confirming cases).

Therefore, the correlation between the treatment and not having gestational diabetes will be the ratio between confirming ( $c$ ) and disconfirming cases ( $d$ ), or  $c:d$ .

- vii. *Multiplicative compensation*: Commonly known as *ratio*, this refers to a relationship between two variables that is constant and multiplicative. This means that if one variable ( $z$ ) increases, the second ( $y$ ) has to decrease in order to maintain the relationship or ratio constant. For example:  $x = yz \Rightarrow 12 = 2 \bullet 6 \Rightarrow 12 = 3 \bullet 4$
- viii. *Equilibrium*: This is similar to multiplicative compensation, but with the particularity that the equivalence has to be maintained between two pairs of multiplicative variables, for example:  $wx = yz$
- ix. *Proportionality*: This is the competence of comparing two ratios and identifying whether or not the relation between them is the same, for example:  $3:15 = 2:10 \Rightarrow 3:9 \neq 2:8$
- x. *Conservation*: This consists of recognising that some things have not changed even if their external appearance has altered. For example, if a plasticine block is converted into a cylinder, that does not mean that the amount of plasticine has also changed. According to Bruner (1959), a child that has not yet reached the formal operations stage is not able to deal with the parts and the whole at the same time. In this sense, if  $A + A' = B$ , when  $B$  is broken into its components ( $A$  and  $A'$ ), the child will think that  $B$  is gone.

Even though Piaget and his co-workers presented evidence for their claims, a group of scholars has claimed that Piaget's theory is old- fashioned (Ennis, 1975, Brainerd, 1978, Corrigan, 1979, Modgil and Modgil, 1982, Halford,

1989, Siegal, 1991) and has either been replaced by other theoretical perspectives or has been rendered unacceptable for explaining cognitive development. With the purpose of presenting an accurate and unbiased picture of the theory that is behind this research and of making my own position explicit, the next section will present the most frequent criticisms of Piaget's work.

### *B. Piaget and his work: criticisms and opportunities*

The fact that Piaget's theory has been criticised and sometimes replaced by newer theories is not surprising. Educational theories are constantly affected by fashion; therefore, theories that were valued a few decades ago are no longer considered or, even worse, are severely criticised and are replaced with new theories that are in fashion. At the moment, socio-constructivist theories are capturing the attention of educators. These theories, according to one of their major exponents, Lev Vygotsky, place a greater emphasis on the social context of learning than does Piaget. By contrast, Piaget has been traditionally understood as being individual-centred, because an important part of his work was devoted to describing the route through which thinking develops from childhood to adolescence, without taking social factors sufficiently into account. Based on this, one of the most common criticisms of Piaget's work is that he deliberately denies the influence of social and contextual factors in human development (Broughton, 1981, Winegar and Valsiner, 1992, Santrock, 1997).

Among education professionals, this discussion is very well known and is presented by DeVries when she argues "Current debate in education on the role of individual and social factors in development often presents Piaget as giving primacy to individual cognitive processes in contrast to Vygotsky's view of the primacy of social and cultural processes" (1997, p.4). However, there is evidence that Piaget's theory did not reject the importance of social factors, with examples that are prevalent throughout his work. For example, Piaget states "Society is the supreme unit and the individual can achieve his inventions and intellectual constructions only to the extent that he is the seat of collective interactions whose

level and value depend obviously on society as a whole” (Piaget, 1971) and “Social life is a necessary condition for the development of logic” (Piaget, 1995).

This misunderstanding of the social aspect of Piaget’s theory is often grounded in the common belief that Piaget’s child is a solitary scientist constructing knowledge apart from the social context (Santrock, 1997). According to Lourenco and Machado (1996), this misinterpretation also has to do with the fact that Piaget’s major interest was trying to identify the emergence and evolution of cognitive development as it happened naturally. In this sense, he established the sequence of cognitive stages that are present in children at different ages and he described the particularities of each of them. As can be seen, although he did not theoretically deny the role of external factors, he did not pay too much attention to the elements that could increase or decrease the speed of appearance of the different stages in his empirical studies.

Piaget’s inclination to empirically describe the construction of knowledge during the lifespan rather than identifying the elements that influence that process is related to the fact that he was essentially a developmental epistemologist and not a social theorist. For this reason, he and his colleagues dedicated long periods of time and great effort to investigating the evolution of knowledge, particularly scientific knowledge. For this purpose, they interviewed individual children using a technique that they called the ‘clinical method’ with regard to a wide variety of problems that involved logical reasoning. According to DeVries (1997, p.4), “When he was concerned with the details of logic in these studies, he did not always mention social factors, and he did not study them systematically. However, throughout his career Piaget also spoke about the development of the child. When he spoke about children development, he always talked about social factors”.

There are other common arguments against Piaget’s theory. One of them is that he underestimated children’s competence as he did not recognise abilities that children had already developed at younger ages than the ones he described (e.g. Halford, 1989). Related to this point, Piaget’s stages have also been criticised due to certain findings that suggest that his age norms are not supported by more recent data (e.g. Ennis, 1975). Based on the description of cognitive development

stages associated with specific ages, some authors have concluded erroneously that Piaget's theory proposed an extremely homogeneous and deterministic sequence of development (e.g. Corrigan, 1979).

With regard to these critiques, it is important to clarify two ideas in favour of Piaget's theory. The first is that, although it is true that in some cases he described children's abilities as accruing at later ages than those at which we now know they develop, this is not proof of an attempt to underestimate children's potential, but rather a suggestion that Piaget's sampling methods might have been inappropriate (or biased), or that his samples were 'less advanced' than the average child. Therefore, even if recent data has not supported the ages at which Piaget described each developmental stage, it does not necessarily follow that the descriptions of the stages are also incorrect.

On the other hand, stating that Piaget's theory is deterministic and homogeneous because it does not take into account individual differences, is clearly a misunderstanding of his theory. He did not argue that each stage has a certain, fixed age for beginning and ending, but that the *sequence* of stages is always the same. In this sense, during his or her lifespan, each person passes through all the stages Piaget described - sensorimotor, pre-operational, concrete operational and formal operational - but at his or her own pace of development, which is certainly influenced by the personal context. In other words, the moment that a person achieves a certain stage in preparation for reaching the next one is influenced by many personal and contextual factors, which make each development path unique.

While criticism of Piaget is often linked to his stage model having assigned ages, in reality, Piaget did not see this as a fixed situation but rather as an approximate reference. When he talks about the appearance of formal operational thinking during adolescence, he states, "The age of puberty varies much less according to climates and civilizations than has been claimed. The age at which the child ceases to feel he is a child and is integrated into the social body varies much more" (Piaget, 1981, p. 61).

As can be seen, various authors have criticised Piaget's theory using a variety of arguments (Brainerd, 1978, Modgil and Modgil, 1982, Siegal, 1991)



and some of them have compared it to and replaced it with Vygotsky's social constructivist approach. At the same time, other authors still value Piaget's theory (Beilin, 1992, Lourenco and Machado, 1996, Papert, 1999, DeVries, 2000, Shayer, 2003, Dawson-Tunik et al., 2004, William M, 2004, Harris, 2009). However, the purpose of this section is not to discuss the position, in favour or against Piaget's theory, that is better supported and why, but to argue that the best position is to complement both perspectives, since this could improve our understanding of human development and learning.

Both theories have undoubtedly made a special contribution to the existent body of knowledge and, from that premise, each should be appreciated and used to illuminate and enrich our educational practice. In relation to this, Cole and Wertsch (2000) argued that we still have many things to learn from both Piaget's and Vygotsky's theories. Similarly, Shayer (2003) states that the complementation of both authors makes a strong contribution to our actual understanding, since the weak points of one is a strength of the other. Similarly, DeVries (2000) agrees that it is necessary to complement both theories through a process that she calls 'reciprocal assimilation' in order to improve our educational practices.

I definitely agree with Cole and Wertsch (2000), Shayer (2003) and DeVries (2000) when claiming that it is best to complement both theories. Reality is extremely complex; thus, it is highly unlikely that a single theory will deal successfully with all aspects of it, which is why each theory usually emphasises or relates to a particular aspect of reality. In addition, the evidence provided by the educational programmes that have taken Piaget's and Vygotsky's principles into account and have integrated them into their practices with a great deal of success (DeVries and Zan, 1994, Adey et al., 2001b) is enough proof of the value of complementing both theories.

Finally, as can be seen throughout this chapter, I definitely feel that even though Piaget's concept of formal reasoning was developed more than 50 years ago, it is still useful for understanding how cognition develops during the human lifespan. In fact, some scholars have adopted and used the concept to explain their findings, although today they refer to it as high-order thinking skills. According to Adey and Shayer (1994), formal reasoning skills and high-order thinking skills

are very similar, because both are essentially cognitive activities that involve complex processes such as analysing data, formulating questions, developing arguments with supported evidence, elaborating opinions, formulating hypothesis, drawing conclusions, criticizing, making decisions and so on (Dori et al., 2002).

However, even though many scholars (Adey and Shayer, 1994, Dori et al., 2002, Barak and Shakhman, 2008) describe the importance of encouraging the development of these types of skills in students, a lot of international evidence that shows that this is not being successfully achieved in many classrooms. The next section will present evidence regarding this issue with the purpose of supporting the relevance of this research project to the current educational context.

### *C. Evidence of lack of formal reasoning skills*

According to Piaget's theory, most adolescents have already developed formal operations (Anderson, 2003). However, more recent research has shown that a significant proportion of secondary schools students (Lawson and Wollman, 1976, Valanides, 1999, Shayer and Wylam, 1978, Lawson, 1978, Lawson et al., 1978, Adey and Shayer, 1990, Shemesh et al., 1992, Adey and Shayer, 1994, Valanides, 1996, 1997a, 1997b) and university students (Renner et al., 1976, Prosser, 1983, Niaz, 1985, Reyes, 1987) have not yet developed these abilities. In this sense, although there is agreement regarding the relevance of teaching high-order thinking skills in schools, this educational endeavour is not being achieved in many countries today.

Research results related specifically to the ability of pre-service teachers to think formally are even more striking. A study conducted by Silverman and Creswell (1982) in the United States concluded that only 14% of the prospective teachers they evaluated showed patterns of formal operational thinking. Similarly, Wyatt (1983) found that only 34% of the elementary majors she assessed were operating at the transitional level between concrete and formal operations, and none of them at the proper formal operational level. In addition, Brownell et al.

(1993) argued that only 7.5% of elementary education student teachers obtained the high formal level score in a formal reasoning test.

This lack of reasoning skills among prospective teachers has led some scholars to stress the relevance of promoting thinking abilities during initial teacher education programmes (Cox, 2007, McDiarmid et al., 1989, Kennedy, 1990, Reynolds, 1992), since these are not only essential for conducting good quality teaching practice, but also for improving these types of skills in their pupils. In relation to this, Peterson (1995, p. 291) states, “Rather than concentrate preservice education on the development of science content knowledge, an alternative approach for improving the ability of preservice teachers to teach primary science is to focus on the development of their pedagogical reasoning ability. Teachers not only need to develop a knowledge base for teaching science, but also need to use their understanding of science content, curriculum and the learner when making decisions regarding their classroom teaching”.

Although most authors agree on the relevance of developing these skills in both teachers and students, they are still not sufficiently emphasised in teachers’ preparation courses (Leat, 1995, Lee, 2005, Barak and Dori, 2009, McDonald, 2010). Taking into account the aforementioned evidence, it is not surprising to find that teachers who are already working in school classrooms experience difficulty in promoting thinking abilities in their students, because most of them did not receive instruction in these skills during their initial training programmes.

### *Some explanations for the lack of thinking skills*

Certain authors (Holt-Reynolds, 1992, Kagan, 1992, Doyle and Carter, 1996) have argued that the weak development of thinking skills is related to the difficulties that teachers experience in promoting these types of abilities in their students. Rather than elaborating a discourse with a large number of mental components, which is a powerful teaching strategy for thinking development, conversation with students is more often limited to giving instructions or explaining content. This conclusion was also corroborated by a study that was

carried out in seven countries, namely Australia, the Czech Republic, Hong Kong, Japan, the Netherlands, Switzerland and the United States of America (Hiebert et al., 2003). In all the participating countries, researchers found that teachers' questions were used more often for pointing out a concept than for explaining or analysing a process. In this sense, what teachers most frequently ask of their students is to carry out mental processes, such as describing or classifying, which do not involve the development of higher abilities like analysis, deductions, hypotheses and inferences.

The results of a study carried out in Chile are consistent with the trend observed internationally. Gonzalez et al. (2008) developed a research project that consisted of analysing teachers' performance inside the classroom in order to evaluate the extent to which their discourse would improve higher thinking skills in their students. They realised that the type of language the teachers used to talk to their students was predominantly orientated towards controlling pupils' behaviour or checking information rather than considering cognitive processes. In this sense, they concluded that teachers do not satisfactorily execute tasks that could encourage their students' thinking skills.

Other studies (Childs and McNicholl, 2007, McLoughlin and Mynard, 2009) have collaborated in constructing the entire picture when concluding that, in traditional classrooms, teachers are the ones who talk most of the time. Apparently, this pattern is not exclusive to certain countries and/or cultures, since the study that took place in the seven countries mentioned above drew the same conclusion, namely "Teachers in all the countries talked more than students, at a ratio of at least 8:1 teacher to student words" (Hiebert et al., 2003, p. 120). Similarly, Mary Budd Rowe (1986) states that primary teachers usually wait one second or even less after asking a question for the students to give an answer, and that they also start speaking again after the student stops speaking. The problem with this situation is that longer waiting times are generally associated with students' progress in terms of their verbal and logical abilities.

Linked to this is the amount of time that teachers leave for pupils' participation and conversation, as the pattern characterises the interaction between the teacher and the students. According to Smith et al. (2004), the most frequent

instructional pattern inside the school classroom is the traditional ‘whole class’ approach. In other words, teachers interact with their pupils as a unit, without fostering individual participation or group work. Therefore, it can be suggested that, although there is a lot of talking inside the school classroom, it may not be used sufficiently for learning purposes, but rather for exposition by the teacher in whole class settings.

The evidence presented so far suggests that teachers are not satisfactorily using discourse as a powerful learning tool that can impact on their students thinking abilities, and they are not giving their students adequate space to participate, to talk and to have discussions inside the classroom. Although these conclusions are already reasonably worrying, this is not the worst of it. A study conducted by Barak and Shakhman (2008) in Israel concluded that most teachers have an ambiguous concept regarding what constitutes high-order thinking skills and only a small group of them think that promoting this type of ability is an important objective of teaching.

### *The relevance of using classroom discourse as a learning tool*

Current psychological evidence suggests that language is one of the most significant teaching tools for improving pupils’ thinking. According to Wells (2002), a major factor in the educational failure that some students experience nowadays is due to the manner in which teachers interact inside the classroom. Teachers’ discourse is characterised by asking closed questions, so the students’ reaction is to try to find ‘the’ right answer. As a result, teachers’ monitoring is clearly oriented towards evaluating the correspondence between their questions and the students’ short answers. These instructional patterns are the opposite of those learning contexts that achieve a multimodal dialogue, in which the main goal is the collective understanding of the subjects and issues discussed (Smith et al., 2004).

With regard to the importance of teachers’ discourse and the development of open-ended dialogues inside the classroom, Bakhtin (1986) proposed what he

called ‘dialogic reasoning’. He states that such types of dialogues are characterised by being creative and multiple, since no one can predict the direction thereof. For Bakhtin, the most important objective of a dialogue is not to reach a common point, but to promote the development of new ideas, grounded in the sharing and discussion of different views.

Alexander (2000, 2005) reaffirms the importance of this type of dialogue inside the classroom, and calls it ‘dialogic teaching’. From his point of view, dialogic teaching consists of promoting students’ participation in the class conversation, in an influential and consistent way, which will facilitate the attainment of better learning results (Mercer, 2008 , 2008a, 2008b). Similarly, Paris and Paris (2001) declared that the educational importance of mental discourse resides not only in the understanding of others’ beliefs, but also in the promotion of self-regulation and, therefore, self-regulated learning in students. In this sense, if teachers elaborate a discourse that contains a high level of mental content, they are more likely to promote the formation of students who are much more capable of understanding their own and others’ opinions (Olson and Astington, 1993).

This perspective is complemented by that of sociocultural psychology (Vygotsky, 1978), which proposes that language has three main functions, namely as a cognitive tool that children use to process knowledge, as a cultural or social tool that allows us to share knowledge and, finally, as a pedagogical tool that allows a person to be another’s cultural guide (Mercer et al., 1999). For this reason, Mercer and Wegerif stated that learning to think is essentially induction into a social practice that involves the internalisation of ‘language as a tool for thinking’ (Mercer, 2000, Wegerif and Mercer, 2000).

All the evidence presented so far indicates that promoting these kinds of skills is a complex and desirable endeavour that all schools should pursue in order to equip their students with the abilities they need in order to succeed. This might suggest that the concept of thinking skills is clearly delimited and well defined. However, that is not the case, as there is much debate among educators and scholars regarding different aspects of it. One of them is the discussion of whether thinking skills are general and can therefore be applied in different contexts, or if

they are specific and subject-related skills that should be developed within a particular curricular area. Given that this research was applied in the context of mathematics, it is relevant to refer to that discussion in order to clarify my position and the kind of assumptions I will make regarding this issue. The next section deals with this discussion.

#### *D. Thinking skills: general or specific abilities?*

Part of the current debate among educators, psychologists, and philosophers is focused on whether thinking skills are general abilities or if they are specific and context-related. Some authors (i.e. Ennis, 1989, Andrews, 1990, Higgins and Baumfield, 1998, Smith, 2002a, Smith, 2002b) advocate the first position and therefore argue that these abilities should be taught in thinking courses. Ennis (2002b) suggests that the existence of general thinking skills is frequently denied because of the misunderstanding that it serves domains, not tasks. According to him, subject-matter domains do not determine the processes through which that domain must be approached. Many domains share tasks that can be solved using the same thinking processes and, correspondingly, many domains involve tasks that have nothing similar about them.

This tradition, which focuses on individual cognition, claims that reasoning processes can be transferred from one context to another and is known as the cognitive perspective. According to Anderson (2000, p. 12), “Cognitive approaches provide analyses about the ways in which knowledge must be structured and about the structures of knowledge in learners’ minds that will be available to support task performance and to transfer to new situations. Additionally, they provide analyses about the kinds of learning experience that will lead to the acquisition of knowledge and skill and its structuring in these ways”. In this sense, cognitive perspectives pay attention to the individual development of cognitive skills and to the transferability of those skills to new contexts.

On the other hand, there is the situative perspective, which “focuses on practices in which individuals have learned to participate, rather than on knowledge that they have acquired” (Greeno, 1997, p. 6). In other words, this perspective focuses on the social aspect of learning and cognitive development. For this reason, situative exponents claim that, for the most part, individual learning is specific to the social context in which that learning was acquired (Lave and Wenger, 1991). Similarly, there are scholars (Barrow, 1987, McPeck, 1990, Barrow, 1991) who refute the existence of general thinking skills by arguing that the range of subjects and objects is immensely wide; thus, there cannot be a limited set of absolutely general thinking skills than can be used to approach such diversity (McPeck, 1990).

Evidently, there is a third group of researchers (Perkins and Salomon, 1989, Niaz, 1994, Moore, 2004) that does not adhere to any of the aforementioned positions, but proposes that both types of thinking abilities are equally relevant. In other words, there is a group of thinking skills that is exclusive of some content areas, and certain others that are transferable and can therefore be used across subjects. For this reason, Niaz (1994, p. 421) states, “this dichotomy is misleading. Instead of being mutually exclusive strategies, the two could very well complement each other”. Even some of the scholars who adhere to one of the two approaches described above (cognitive or situative perspectives) state that both research lines should be combined in order to strengthen current understanding of education and learning (Greeno, 1997, Anderson et al., 2000).

As I have tried to describe in this section, it is impossible to state with absolute certainty whether thinking skills are general or specific, because the debate has not been concluded. My personal inclination is closer to that of scholars who claim that once a person has developed and consolidated a specific thinking skill, that skill will be useful in different contexts. However, as I am not an expert on the topic, I am not able to refute the arguments raised by those who adhere to the “specific” perspective. Nonetheless it is relevant to remark that, even taking these differing points of view into account, most scholars agree (Torff, 2003, Preiss and Sternberg, 2006, Barak and Dori, 2009) that improving thinking skills, in either their general or specific formats, is a very important aim of current educational systems.



The recent interest in thinking skills in educational settings has arisen in reaction to technological advancement, whereby content and knowledge are not only more accessible than they were for previous generations, but ideas and perspectives are also evolving more swiftly. This renders teaching techniques that are focused on the transmission of information outmoded, because when information or content changes quickly, teachers have to focus on developing the students' skills that will permit them to find, manage, select, criticise and update that information. According to Tal and Hochberg (2003), the modern world forces people to participate in diverse and constantly changing environments; thus, they require these skills in order to succeed. Therefore, educational professionals have the responsibility of encouraging their students to develop a wide range of strategies that permit them to use the increasing amount of knowledge and experience, and to search for new and innovative ways of solving problems.

In this context, it is possible to see why there is currently a strong call for educational systems to reform their schools "from teaching basics skills towards schools for thought" (Zohar, 1999, p. 413). One of the programmes that have done this successfully is the Cognitive Acceleration approach that was designed and implemented for the first time in the UK. From my point of view, the value of this approach is not only that has effectively promoted the development of thinking skills in UK classrooms for the last 30 years, but also that it has encouraged students to make connections between what they have learned in the classroom and real-life contexts. In addition, Cognitive Acceleration is not only a teaching and learning approach, but is also a rigorous line of research that has kept records of the impacts of the programme. Therefore, I chose this approach from among all the others that deal with this issue (Siegler et al., 1973, Case, 1974, Kuhn and Angelev, 1976, Lawson and Blake, 1976, Lawson and Nordland, 1976, Kuhn et al., 1979, Rosenthal, 1979, Feuerstein et al., 1980, Lawson and Snitgen, 1982, Rosenshine, 1992, McGuinness, 2000, Panizzon and Bond, 2007) for this research project. The next section will describe the original programme that was designed in the context of science education, followed by the one used for this specific project, mathematics.

## Cognitive Acceleration Programmes

### *E. Cognitive Acceleration in Science Education (CASE): General Description*

Piaget described the process through which a child develops higher levels of thinking in terms of the interaction between the individual and the environment. When the child faces a new situation, there are two possible ways of dealing with it. The first is to integrate the new stimuli to the existing cognitive structure, and is called *assimilation*. The second scenario is that the previous cognitive structure could not assimilate the new stimuli. In that case, the structure needs to change through a process called *accommodation* or *equilibration* (Piaget, 1964).

In this sense, the stimuli present in the child's environment play a crucial role in his or her cognitive development by challenging the current cognitive structures and by forcing the child to adapt to new and more complex organisational forms. Accepting this statement could lead to the understanding that the teachers' role is being in charge of placing or presenting the correct stimulus to promote the students' thinking. The key characteristic of these stimuli is that they have to be sufficiently challenging to produce the necessary cognitive instability.

During the early 80s, this assumption encouraged Michael Shayer and Philip Adey (1994) to develop the first Cognitive Acceleration project, namely Cognitive Acceleration through Science Education (CASE). In general terms, the programme consisted of creating and using different thinking activities instead of ordinary science lessons in order to promote and enhance students' thinking abilities in terms of the type of thinking that Piaget called 'formal operations'. Although the literature describes a wide range of cognitive acceleration initiatives (Siegler et al., 1973, Case, 1974, Kuhn and Angelev, 1976, Lawson and Blake, 1976, Lawson and Nordland, 1976, Kuhn et al., 1979, Rosenthal, 1979, Feuerstein et al., 1980, Lawson and Snitgen, 1982, Rosenshine, 1992, McGuinness, 2000,

Panizzon and Bond, 2007), very few of them lasted more than two months and had the purpose of training general thinking abilities that could be transferred to other tasks, situations or domains.

In this scenario, the particularities of the CASE project were certainly original. The intervention was designed to be delivered over two school years and was intended to promote higher order thinking skills in students who participated in the programme. These activities were designed to enhance students' thinking capabilities as a whole, not only in a science context. The underlying assumption of this approach is that there is a general thinking processor, according to which natural development can be accelerated through specific intervention (Adey, 1999).

### *CASE pillars*

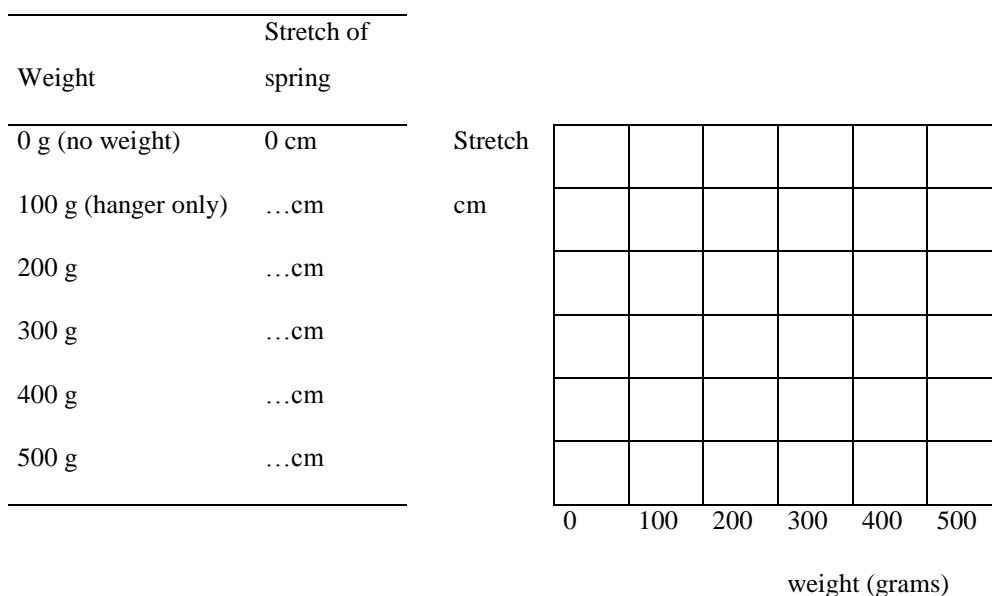
The CASE intervention is a well-defined and structured teaching method based on Piaget and Vygotsky's theories. Thus, cognitive acceleration (CA) methods have three main pillars. The first pillar, cognitive conflict, comes from Piaget's theory. As described earlier, he thought that humans require cognitive balance, or equilibration. In this sense, when a child faces a situation that poses a problem that s/he cannot solve with his or her existing cognitive structures, s/he experiences cognitive conflict that needs to be resolved through assimilation or accommodation in order to regain equilibrium. In support of this theory, CA confronts students' present assumptions and ways of thinking through the presentation of a problem, with the purpose of promoting cognitive development. This does not mean that extremely difficult activities are used, but that they operate at an appropriate level in order to challenge students' current assumptions.

As an illustration of cognitive conflict produced by the presentation of appropriate information, I will explain a cognitive acceleration activity called 'What sort of relationship?' (Adey et al., 2001b, p.11). During the lesson, two experiments are conducted in which one variable is related to another. The first experiment consists of using a graph to represent the relationship between spring

extension and weight (see Figure 5). For this purpose, students need to suspend objects with different weights and to register the extension of the spring in centimetres. Year 7 children's finding will match their expectations; in other words, a linear and direct relationship between the two variables, whereby the heavier the object is, the longer the spring will be (Adey et al., 2001b, p.11).

The second part of the activity consists of developing a second experiment that entails finding out how long it takes for a certain amount of oil to pass through a funnel at different temperatures. This time, the relationship between the variables will not be direct and linear, but curvilinear and inverse since, as the oil temperature increases, the time taken for it to pass through the funnel decreases. This will cause cognitive conflict in students and the teacher will have to promote group discussions in order to hypothesise causes and to find the right explanation for this phenomenon (Adey et al., 2001b, p.11).

**Figure 5: CASE activity example**



(Adey et al., 2001b, p.17).

The second pillar is social construction that comes from Vygotsky's theory. He described learning as a process that is pre-eminently social, and in which older people or peers play a crucial role. Vygotsky developed the concept of the Zone of Proximal Development, which is "the distance between the actual

developmental level as determined by independent problem-solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers” (Vygotsky, 1978, p.86). In this sense, children’s cognitive development or mental growth is essentially a social process. CA adopted this theory and applied it to the activities designed. For this reason, group work is one of the most important characteristics of any CA lesson, because it gives students the opportunity to create and to share knowledge.

The last pillar is metacognition, which means thinking and reflecting on both the individual and the group’s thinking processes. This pillar originates from the idea that children should come to think of themselves as ‘thinkers’ who have some control over their own thinking process. The ability and the disposition to reflect on how one has solved (or even failed to solve) a problem is a powerful tool that enables children to take more control of their own learning (Adey, 2008, p.12). There are also two complementary pillars, namely concrete preparation, which is the first part of the lesson in which the teacher introduces the topic, and bridging, which consists of finding everyday situations or other contexts in which the students could apply the type of thinking practised in the lesson (Adey, 2005). These two pillars support the thinking processes in which the learners are involved during the cognitive conflict, construction and metacognition phases. The concrete preparation step equips the learners with the necessary vocabulary and actions to begin considering the problem, while bridging helps the learner to recognise the features of the reasoning pattern within a different context.

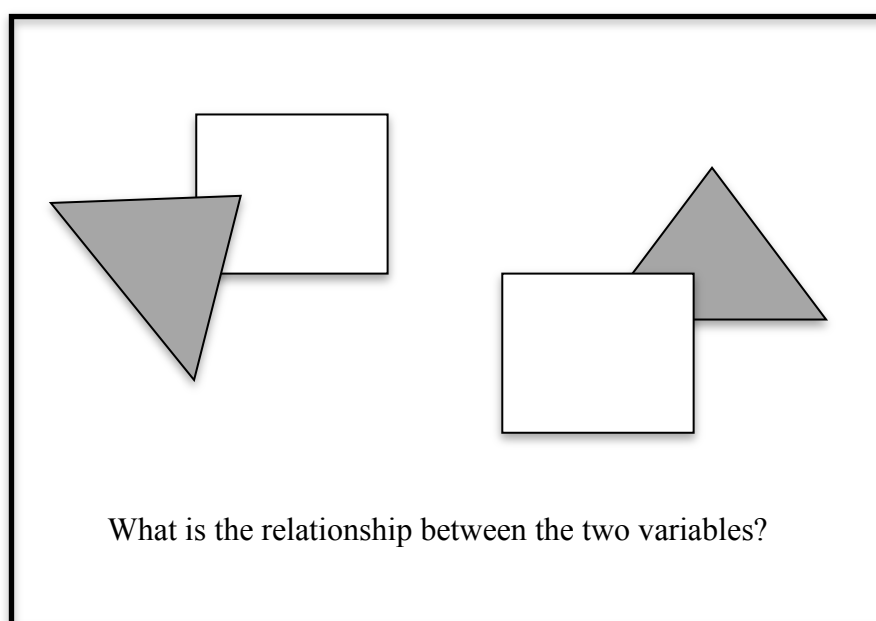
### *CASE lesson structure*

All these pillars have been integrated into the teaching method that characterises each activity of the project: the Three-act Model. Act 1, the first part of the lesson, is called concrete preparation, in which the teacher interacts with the class as a whole, introducing the necessary terms to think and talk about the problem. For example, in Act 1 of the activity ‘What varies?’, the teacher needs to introduce the terms ‘variable’, ‘value’ (as it applies to variables) and ‘relationship’, together with the possible relationship between two variables

(Adey et al., 2001b, p. 6). For this purpose, the teacher presents the class with an image, such as the following (see Figure 6) and, given the relationship, asks the students to predict colour from shape and shape from colour. Following this discussion, students must understand the concept of the variable as the thing that can vary and the relationship as a means of predicting multiple characteristics when given one data point.

In Act 2, the class is divided into smaller groups (between 2 and 5 students), in which pupils can discuss and exchange ideas that allow them to have certain insights about a presented problem. This part of the lesson is one of the most important, not only because students work collaboratively and construct shared knowledge, but also due to the experience of cognitive conflict. Cognitive conflict can occur spontaneously when students realise their ideas are insufficient for producing a solution, or because the teacher intervenes by raising questions that make them realise the contradictions of their ideas.

**Figure 6: What varies?-Act 1**

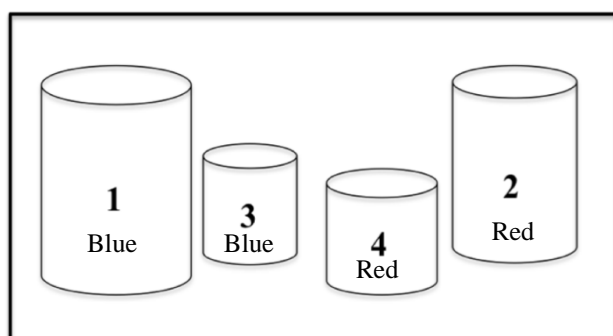


(Adey et al., 2001b, p.1).

During Act 2 of the 'What varies?' activity, the teacher presents the pupils with a group of sealed containers (see Figure 7) and asks them to complete the table with the variables and to find the relationship between them (see Table 5).

To the students' surprise, there is no relationship between size and weight. Pupils' expectations from their everyday experiences is that large containers are heavy and small containers are light but, in this investigation, containers 2 (large) and 4 (small) are heavier than are 1 (large) and 3 (small). This finding is very important because, in order to understand the notion of relationship, they must encounter examples with no relationship. After this, pupils will keep working in groups with work cards specially developed for these purposes and, for each picture, they should determine the variables (what are the things that vary?), the values of each variable and whether there is a relationship between the variables. If so, what sort of relationship is it? (Adey et al., 2001b, p.7).

**Figure 7: What varies?-Act 2**



(Adey et al., 2001b, p.1).

**Table 5: What varies?-Pupil's table**

Beaker number	Colour	Size	Weight
1	blue	large	150 g
2	blue	large	250 g
3	red	small	150 g
4	red	small	250 g

(Adey et al., 2001b, p.7).

During Act 3, the teacher conducts a whole-class discussion about their findings once s/he realises that all the students have constructed sufficient ideas to understand the problem, even if they have not yet solved the entire problem. Metacognition occurs in this part of the lesson when the teacher asks the class to describe the problem and the students see or hear classmates whose thinking has gone much further than has theirs. The teacher will frequently ask the pupils, ‘How did you solve the problem?’ or ‘Please explain to the others in your group why you think that’. An important part of the trick of developing thinking skills is for students to become conscious of and articulate about the type of thinking they are employing to solve different problems. Thinking back and reflecting aloud helps to develop this consciousness (Adey et al., 2001b, p.5).

Finally, there is an optional Act 4, in which the teacher tries to lead the students to think about what they have learnt and how this could be useful in other contexts (Adey and Shayer, 2002). Without this sort of mental bridging back and forth between the activities, there is a danger that they will be seen as ‘peculiar’ or ‘special’, and that any effects that may be achieved will remain specifically linked to certain types of activity and will not be generalised to science or, even better, beyond science (Adey et al., 2001b, p.5).

### *The teacher’s role in CASE lessons*

When applying these pillars in the classroom, the role that teachers play in the successful implementation thereof is crucial. They do not have to focus on the transmission of information or content. In contrast, they have to give the students the opportunity to engage in higher-order thinking processes through thinking hypothetically, challenging their assumptions, supporting them rationally and discussing different views with their classmates (Adey, 2005). Given the particular characteristics of the method, it is fundamental that teachers manage to develop a safe classroom climate characterised by respect and collaboration. Only



in this type of atmosphere can every student feel comfortable and confident in engaging in the cognitive challenges that the teacher provides for them.

In accordance with Adey (1999, p.25), “To create such an atmosphere, teachers need to have clear objectives in terms of the type of reasoning being developed in a particular thinking lesson; some familiarity with the underlying theory of cognitive acceleration; an intimate understanding of the range of reasoning and arguments displayed by his or her pupils, if not of the particular levels of argument employed by each individual pupil; mastery of a range of techniques such as asking leading questions, suspending judgement, setting challenges appropriate to particular children”. As can be expected, teachers are usually required to change many of their practices and to develop new ones in order to successfully implement the programme. Therefore, CASE always starts with an extensive in-set preparation stage for teachers because, otherwise, the time devoted to the intervention as a whole might be unfruitful.

### *CASE programmes' effectiveness*

Most cognitive acceleration programmes developed in research contexts have had a quasi-experimental design in which all students have been administered pre-, post- and delayed post-tests (one year after the end of the intervention programme) in order to compare the experimental group's results with the ones obtained by the control group. In the following tables, it is possible to observe the design, the tests used in the original CASE intervention (see Table 6) and the gain scores (see Tables 7 and 8), or residualised scores, of the control and experimental groups. These tables illustrate that the students assigned to the experimental condition showed statistically greater cognitive development after the CA programme than did their control counterparts.

**Table 6: Experimental Design for Original CASE Research**

	Date	Experimental Group	Control Group
<b>Pre-Tests</b>	Sept. '85	2 Tests of levels of cognitive development	
<b>Intervention</b>	Sept. '85- June '87	CASE intervention	Normal science classes
<b>Post-Tests</b>	June '87	(i) 2 tests of levels of cognitive development (ii) 1 science test	
<b>Delayed Post-Tests</b>	June '88	(i) 1 test of levels of cognitive development (ii) 1 science test	
<b>GCSE**</b>	June '89 or '90*	GCSE grades in (i) science (ii) mathematics (iii) English	

(Adey, 2005, p.5).

**Table 7: Laboratory School Scores in Piagetian Reasoning Tasks**

Group	N	Test and date				Gain scores: pre to...	
		Pre-	Mid-	Post-	Delayed	Post	Delayed
		Jan-84	Jul-85	Jan-86	Jul-87		
Boys and girls							
Experimental	29						
M:		5.89	6.46	6.35	7.01	0.46**	1.13*
σ:		0.55	0.72	1.00	1.21	1.95	1.12
Control	19						
M:		6.46	6.12	6.26	7.01	-0.2	0.54
σ:		0.36	0.56	0.87	1.09	0.7	0.86

**Boys**

**Experimental** 15

<b>M:</b>	5.84	6.52	6.35	7.03	0.51	1.19
<b><math>\sigma</math>:</b>	0.59	0.88	1.14	1.48	1.17	1.36

<b>Control</b>	11					
<b>M:</b>	6.54	6.81	6.64	7.27	0.1	0.73
<b>σ:</b>	0.41	0.53	0.89	1.05	0.69	0.71
<hr/>						
<b>Girls</b>						
<b>Experimental</b>	14					
<b>M:</b>	5.94	6.41	6.35	7	0.42**	1.06*
<b>σ:</b>	0.52	0.59	0.87	0.86	0.92	0.83
<b>Control</b>	8					
<b>M:</b>	6.35	6.53	5.15	6.64	-0.61	0.29
<b>σ:</b>	0.26	0.56	0.55	1.09	0.49	1.02

(Adey and Shayer, 1990, p.273)

\*Significantly greater than control at  $p < 0.05$ .

\*\* Significantly greater than control at  $p < 0.01$ .

Although the results from Table 7 confirm that the intervention was successful, the authors were also looking for a relationship with both academic achievement and far transfer evidence (other school subjects). As a result, they also compared experimental students with control ones in terms of their achievements in the General Certificate of Secondary Education (GCSE) in science, maths and English. Table 8 shows that experimental boys (Y8) and girls (Y7) performed better than did their control counterparts in the three GCSE tests. The fact that the results were especially noticeable for experimental girls, who benefited most from the intervention after two years, was a particularity observed only in the original CASE intervention. In other words, that was the only occasion when the results for girls were much better than were those of boys participating in the intervention.

**Table 8: Effect Sizes from the Original CASE Experiment.**

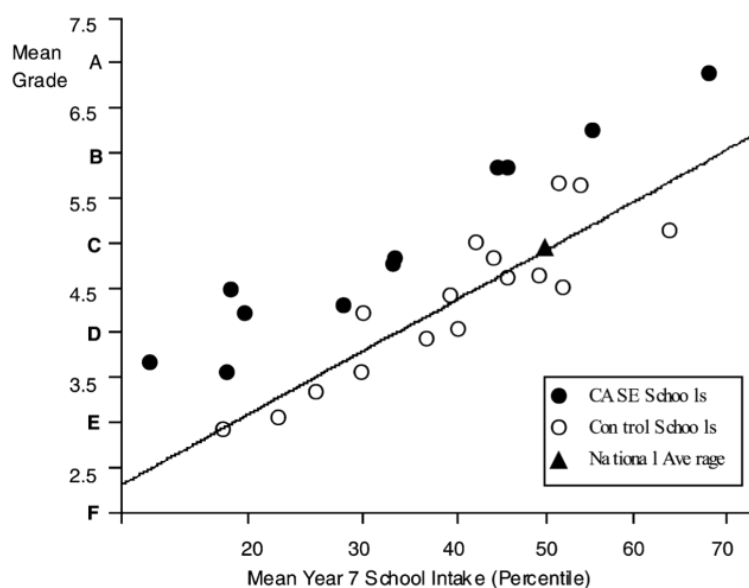
		Immediate post '87		Delayed post	GCSE, '89 or '90		
		Cog. Dev.	Science	'88 Science	Science	Maths	English
<b>Year 7</b>	<b>Girls</b>	-	-	0.60	0.67	0.72	0.69
<b>start</b>	<b>Boys</b>	-	-	-	-	-	-
<b>Year 8</b>	<b>Girls</b>	-	-	-	-	-	0.44
<b>start</b>	<b>Boys</b>	0.75	-	0.72	0.96	0.50	0.32

(Adey, 2005, p.7)

Only statistically significant ( $p < 0.01$ ) effect sizes are shown. All effects are of CASE/non-CASE.

The results reported so far are from the first CASE intervention. Following the original programme, the authors changed the design of the study in the sense that they no longer compared individual students or classes, but entire school units. The reason behind this shift was that, from an ethical point of view, it was not possible to have a control group because that would have implied voluntarily leaving a group of students without the possibility of benefiting from the treatment. As a result, they started to compare CASE schools with similar non-CASE schools using the value-added approach.

**Figure 8: Grade comparison between CASE and non-CASE schools**



(Adey, 2005, p.9).

By using the value-added model, the authors again found that CASE intervention had a long-term and far-transfer effect on experimental schools, which means that it was possible to observe improvements years after the intervention was complete, and that these improvements were also observable in subjects other than Science. Figure 8 shows that not only are most CASE schools above the national average, but that they also have higher mean grades than do the control or non-CASE schools.

Since from the beginning of the implementation of cognitive acceleration programmes, Shayer (1996) and Adey (1999) have demonstrated that CASE activities and approaches have a very strong impact, both in terms of students' thinking ability and on their academic achievement, even in other school subjects such as mathematics and English (Adey and Shayer, 1994, Shayer, 1996, Adey, 1999, Shayer and Adhami, 2003, Adey, 2005, Iqbal and Shayer, 2000, Adey et al., 2002). This evidence supports the assumption that there is a general processing capacity that can be affected through the intervention. Based on these results, Adey (2005) has suggested that students' thinking skills that are enhanced by the CASE programme are not limited to science, but could be applied to any subject, thereby improving the children's grade attainment in all subjects.

Due to the Piagetian foundation of the CA programme, the two-year intervention was originally developed for 11-14 year-old children. According to Inhelder and Piaget's (2003) findings, most adolescents have already developed formal operational thinking. However, Shayer and Wylam (1978) found evidence that contradicted the previous findings. They conducted a large-scale survey among British adolescents using science-reasoning tasks that they had developed from a Piagetian perspective. Based on these tasks, Shayer and Wylam (1978) concluded that only about 5 per cent of 11 year-old children exhibited early formal operations. In this sense, implementing an intervention at these ages could have a critical impact on children's future thinking development.

Following the successful trials of the CASE intervention in England, Shayer began to look at developing similar ideas in other classroom subjects, and developed a similar intervention project for mathematics called CAME (Cognitive Acceleration in Mathematics Education). He used a similar approach to the one he

and Adey had developed for CASE, utilising the original principles, assumptions and theories that motivated the implementation of CASE. However, CAME is a cognitive acceleration programme set in the culture of a different subject, namely mathematics instead of science, was implemented for the first time more than 10 years after the original CASE and exhibits some differences in approach. Therefore, the next section will describe the approach and the activities, and the ways in which teachers are prepared to teach CAME, and will also present evidence regarding the impact that CAME has had on the participating students.

### *F. Cognitive Acceleration in Mathematics Education (CAME) Project*

#### *Origins and general description*

While the CASE project has been implemented since the 1980s, CAME was delivered for the first time in 1993. The original intention was to foster cognitive development in secondary school students who were 11 to 14 years-old or, in other words, to encourage students to think mathematically (Shayer et al., 1999). The relevance of developing a project with that purpose was similar to the one held by CASE in its beginning stages; the understanding that a large part of the mathematics school curriculum demands the use of formal reasoning skills in order to comprehend it in-depth, and the evidence that showed that only 20% or 30% of 14 year-old students had already developed these reasoning skills (Shayer and Adhami, 2007).

According to Shayer and Adhami (2007), the situation was aggravated by the wide spread belief among adolescents that abilities are explained by genetic potential, in that one is either good or poor at mathematics, thus suggesting that an intervention for cognitive acceleration was not a feasible possibility. For this reason, a plausible option to counteract this situation was to carry out a school-based intervention that could promote or accelerate the transition from the

concrete operational stage to the formal operational one, in order to demonstrate to the students that they were able to achieve good results in maths.

To accomplish these objectives, the CAME project provides a set of 30 activities designed to be carried out four or five times each school term over a two-year period. Each activity demands that students organise conceptual strands in mathematics, instead of using just procedures and algorithms as they would do in 'normal' mathematics lessons. In other words, rather than promoting a mechanical or memory-based way of solving problems, CAME attempts to develop reasoning skills thorough the process of requiring students to reconstruct the underlying mathematical concepts and the reasons for them (Adhami et al., 1998).

All the activities were designed and adapted via the following process. Firstly, they were piloted by the authors in one experimental class over two school year periods. Based on that experience and on what the authors learned from it, the activities were adapted and improved. Having done that, they gave the newly adapted activities to three different school teachers who were participating in the process, and one of them observed the development of each lesson in order to take notes. Finally, field notes from the CAME lesson observation were used for developing a preliminary version of the programme that was to be trialled in 14 school classrooms with students in years 7 and 8 (Adhami et al., 1998).

It is important to remark that there was one very important difference in the process of constructing CAME activities in comparison with those of CASE. According to Shayer and Adhami (2007), mathematical language is very powerful and suggestive, leading students to develop certain mechanical algorithms and to obtain correct results even when they did not know why or what they were doing. For this reason, there were two lines that guided the design of each CAME activity, as follows:

- i. All the problems and concepts were placed in an everyday context in which a range of achievement levels was possible, in order not only to allow all students to participate and contribute during the lesson, but also to make improvements from their own starting points.
- ii. The essence of each lesson was based on creating a context in which

individual and group constructions were the key. In this sense, pupils were not only encouraged to develop procedures and algorithms, but also to be challenged to articulate the reasons behind them. A further point was for students to work out how the topics were connected to each other.

### *CAME lessons*

CAME lessons, similar to CASE lessons, are based on collaborative activities that utilise dialogue to stimulate and promote high order thinking. Each CAME activity lasts between 60 and 90 minutes and the teacher acts as a mediator during group and class exchanges. In this sense, although each CAME activity uses mathematical concepts for promoting students thinking skills, the lessons do not deal with them directly by exposing the concepts, but indirectly through individual, small group or whole-class work. Given the particular characteristics of CAME activities, they are not intended to substitute regular school lessons but to complement them, since students are given the opportunity to both learn and to investigate at the same time (Adhami et al., 1998).

The set of CAME activities covers the entire range of reasoning patterns described by Piaget as formal operations, under the form of nine content strands derived from the UK National Curriculum for Mathematics. Table 9 presents the content strands range addressed by the set of secondary CAME activities. As can be seen, each lesson is oriented to deal with one main strand (represented in Table 9 as solid black circles) and other secondary strands (represented in Table 9 as empty circles). As a whole, the full set of activities will address the curriculum in a spiral sequence (Shayer and Adhami, 2007).

Given the variety of abilities present in every classroom, CAME activities try to adapt to students who are performing at two or three different levels. In other words, this feature allows challenges to students' current assumptions, and therefore promotes learning even if they have different developmental levels. Each activity is not only focused on one of the strands shown in the table above,



but also addresses other secondary strands. Table 9 shows the main and the secondary strands dealt with in each lesson and the range of abilities covered, from preoperational stage 2A to formal operational 3B. (Shayer and Adhami, 2007)

In terms of the structure of the lesson, activities at the beginning introduce a familiar context in order to make sure that all students have the necessary background for understanding and developing the other parts of the lesson. Students then work on some of the mathematical problems that CAME provides. In order to solve these challenges, students will need to accommodate their thinking patterns to higher levels. This may not happen spontaneously, so CAME teachers will have to guide students through the problem by raising questions that encourage students to solve it (Adhami et al., 1998).

**Table 9: The Secondary CAME Thinking Maths Lessons by Strands**

Suggested order	Strand Code	Lesson and main NC levels	Number system and properties	Multiplicative relations	Functions	Algebra models	Shape and space	Data handling	Range of levels of expected outcomes in episodes			
									Piagetian and NC levels			
									2B 4	2B* 5	3A 6	3B 7/8
1	A1	Algebra	4-7		●	○	○					
1a	A1a	Algebra	4-6		●	○	○					
2	A3	Text'n'talk	4-6		●	○						
3	N1	Number lines galore	4-6	●	○		○					
4	D1	Furniture design	4-6	○			○	●				
5	D2	Sam and the newspaper	4-6					●				
6	S1	Decontamination	4-6	○			●					
7	A4	Which offer shall I take?	4-6		○	●	○					
8	N2	Ladders and slides	4-7		●	○	○					
9	S2	Framed tiles	4-6		○	○	●					
10	S3	Rectangle functions	5-7		○	○	●					
11	N3	Setters and solvers	4-6	●	○							
12	N4	Functions	5-7		●	○	○					
13	A5	Chocolate box	5-7		○	●	○					
14	S4	Tents	4-7		○		●					
15	S5	Circle functions	5-7		○		●					
16	D3	Three dice	5-7		○			●				
17	N5	Sets and subsets	4-7	○	●		○					
18	D4	Prediction and correlation	5-7					●				
19	N6	Accuracy and errors	5-7	●	○							
20	D5	Heads and tails	5-7					●				
21	A6	Expressions and equations	5-7	○		●						
22	D6	Comparing correlations	5-7		○			●				
23	A7	Rates of change	5-7		○	●	○					
24	D7	Data relations	5-7	○				●				
25	S6	Triangle ratios	5-8		○		●					
26	A8	Chunking in algebra	5-8			○	●					
27	A9	Accelerating the acceleration	5-8			●	○					
28	S7	Graph of the rotating arm	5-8		○	●	○					
29	A10	Straight line graphs	6-8		○	●	●					
30	D8	How do I handle the data?	5-8		○	○		●				

● Main strand focus

○ Other strands covered

(Shayer and Adhami, 2007, p. 11).

### *Learning to teach CAME*

As in any other cognitive acceleration programme, teachers play a crucial role in CAME lessons. Teachers need to challenge students and to encourage them to work collaboratively in order to solve each of the problems. As this usually entails teachers developing new classrooms practices, teachers are asked

to participate in a professional development training before implementing CAME programmes. These programmes will hopefully equip teachers with the tools and skills they need to become confident and capable CAME teachers.

Shayer and Adhami (2003) state that, from the beginning, it was difficult to design the professional development for CAME teachers because the trainers needed to introduce a completely new way of teaching mathematics, which contradicts the commonly held belief that mathematics is a given set of facts, rules and algorithms. If they wanted to create a suitable context for promoting students' thinking abilities, they "needed to revise mathematics teaching in the direction of treating mathematics as if it were an ill-structured discipline...we needed to take seriously, with and for young learners, the propositions that mathematical statements can have more than one interpretation, that interpretation is the responsibility of every individual using mathematical expressions, and that argument and debate about interpretation and their implications are a normal part of mathematical activity" (Adey and Shayer, 1994).

Therefore, the professional development for CAME teachers takes into consideration both Piaget's cognitive development and Vygotsky's social construction as a background. If the training stage is to promote new ways of thinking and teaching maths, teachers should experience the same learning environment that they are supposed to create in their own classrooms. "If their students are being asked to construct their learning through a collective and collaborative process then it follows that teachers cannot simply be told how to do it. They need to experience a comparable process in their professional development" (Shayer and Adhami, 2007, p.288). In this sense, teachers do not need to acquire a set of skills, but to internalise and share the cognitive and social agenda that each CAME activity requires.

For this reason, the professional development of CAME teachers requires teachers to work in small groups when solving certain tasks or problems, and to then share their views and conclusions in whole class discussions. Other activities include designing new lessons, testing them, observing each other's teaching process and giving feedback. As can be seen, CAME training is not a prescriptive process that presents a limited number of activities and ways of teaching them. In

contrast, it needs to be a constructive process that promotes new ways of teaching and thinking about teaching and, at the end, one that empowers teachers to create new activities, adapt them for students at different levels and adapt the approach for other learning environments.

In this sense, as teachers are not only individuals but are also parts of bigger units (departments and schools), Adhami et al. (1998, p. viii) proposed that the development of professional communities is the most effective way of generating shared knowledge and meaningful experiences for the relevant group of students using CAME activities. Therefore, the authors claimed that teachers should work collaboratively and that collaboration may be accomplished by following three main steps, as listed below:

- Share the work of making a detailed plan and a time-line for each CAME activity, tailoring the activities in the book to their particular pupils.
- Arrange to see how pupils respond to the plan in each other's lessons.
- Have a departmental de-briefing session - at least for the first five CAME lessons – for collaboration in developing the teaching approach (Adhami et al., 1998, p. viii).

### *CAME impacts and results*

During the first two years of the CAME project, three schools participated in the piloting and design of the lessons. In each school, one class was taught by the head of the mathematics department. After the first two years of the trial, 11 schools were chosen to participate and all their Year 7 students took part in the study. The researchers divided the schools into three groups (Shayer and Adhami, 2007):

- i. 'Core' schools, which had experimental classes and their teachers received frequent, in-school training by the research team
- ii. 'Attached' schools, which also had experimental classes, but their teachers had to attend to the training sessions held at King's College

London

iii. Control classes

In order to assess the impact of CAME, all the students from the experimental and control classes took a mathematics test at the beginning (pre-test) and at the end (post-test) of the two-year intervention. In addition, in order to evaluate the transferability and permanence of the impact, students' results in the General Certificate of Secondary Education (GCSE) for maths, science and English at the end of Year 11 were included in the study (Shayer and Adhami, 2007).

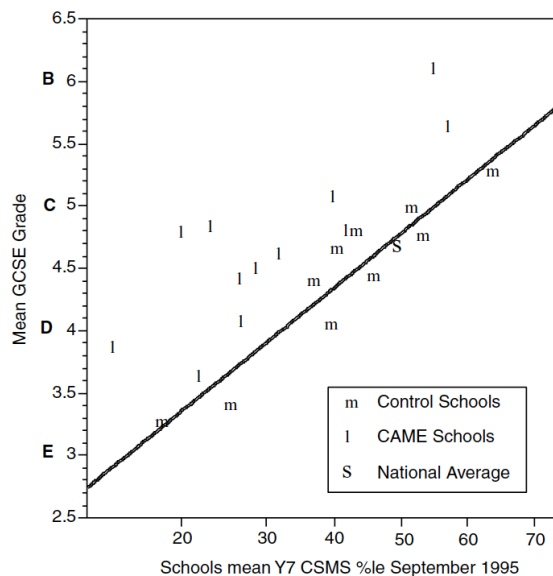
As Table 10 shows, the results obtained by the experimental classes immediately after the intervention was over are not very impressive in terms of effect sizes. Effect size is the difference between the pre- and the post-test, expressed in units of standard deviation (Adey, 2005). However, when the data from the control groups and from the GCSE are included, the picture changes (See Figure 5). For calculating the value added by CAME in terms of GCSE grades, each experimental school's mean grade in the GCSE is plotted against the mean obtained by the same school in the pre-test taken at the beginning of Year 7. Similarly, the regression line for control schools is drawn based on their GCSE grades. Consequently, the distance between the regression line and each school's mean is the value added by CAME to GCSE grades (See Figure 9) (Shayer and Adhami, 2007).

**Table 10: Pre- and post-test school means on the maths test**

Post-test					
School	Pre-test	Predicted	Obtained	Effect (SD)	P
Core 1	6.08	6.49	7.00	0.41	<.01
Core 2	5.32	5.79	6.02	0.18	<.05
Core 3	5.03	5.52	5.66	0.13	n.s.
Core 4	5.45	5.91	6.47	0.52	<.01
Attached 1	5.63	6.08	6.58	0.49	<.01
Attached 2	5.99	6.41	7.02	0.56	<.01
Attached 3	4.77	5.29	5.59	0.28	<.01
Attached 4	5.69	6.13	6.15	0.01	n.s.
Attached 5	5.30	5.78	6.17	0.38	<.01
Attached 6	5.29	5.77	5.97	0.2	<.025
Attached 7	5.68	6.13	6.76	0.62	<.01
Overall mean				0.344 SD	

(Shayer and Adhami, 2007, p.278).

**Figure 9: Added value in GCSE 2000 maths for CAME schools**



(Shayer and Adhami, 2007, p.280).

Based on the results observed three years after the intervention, the researchers accept that it is not very likely that the set of CAME activities itself is the only explanation for the students' progress. In contrast, Shayer and Adhami (2007) believe that what CAME does is to encourage and promote the professional development of the teachers who participate in the intervention. It was explicitly suggested by the trainers that teachers needed to make connections between the cognitive aims of the CAME activities and their teaching during regular mathematics lessons. As a result, teachers started to make use of the same teaching skills across the curriculum and students took an approach that was more focused on thinking during their learning. This may have produced a virtuous circle that, in the end, had a positive impact on students' cognitive development and learning.

Even though those explanations and hypothesis are plausible, others scholars (Leo and Galloway, 1996, McLellan, 2005) have used different arguments to explain the differential effects that CA programmes have had on the participating students (Adey and Shayer, 1990). For that reason, the next section will present some of the most important alternative explanations for CA results in order to give a better picture of the limitations of CA research.

### *G. Cognitive Acceleration programmes: Alternative explanations*

Because the cognitive acceleration programme produced differential effects on the students that participated in the intervention (Adey and Shayer, 1990), some authors (Leo and Galloway, 1996, McLellan, 2005) have tried to explain these differences by considering why CA has had a positive impact on some students' thinking skills and not others by referring to motivational theories.

According to Leo and Galloway (1996), every child's learning experience is necessarily mediated by his/her motivational style. As a result, the motivation to learn will constitute a positive or negative predisposition to face any learning situation that is completely independent of the child's real aptitude. This means

that the motivational style is a psychological tendency that has an important effect on every child's learning and is not related to what the child is actually able or not able to do, but to what he/she believes s/he is able to do. The authors refer to the three major motivational styles described by Dweck et al. (1995):

- i. *Learned helplessness*: A learned helpless style is characterised by the belief that failure is inevitable and, for that reason, the child tries to evade any learning situations that are challenging and stops trying quickly, because he/she is convinced that his/her effort has no effect on performance.
- ii. *Self-worth motivation*: This style tries to maintain a positive self-concept no matter what. As a result, if the child faces something challenging, s/he prefers to give up trying by claiming that the task is boring or not important in order to be able to protect his or her self-esteem because, if the child tries and fails, the conclusion would be "I'm not able" or "I have a low ability".
- iii. *Mastery oriented*: Children that have this motivational style are concerned with learning and with facing challenging situations that will make them progress. They perceive effort as a tool for learning and they do not feel threatened when experiencing difficulties, because they care more about learning than about ability.

Based on this categorisation of different motivational styles, Leo and Galloway (1996) claimed that mastery-oriented children are more likely to respond positively to CA lessons for several reasons related to the structure thereof. Firstly, as *concrete preparation* requires children to share their prior knowledge and ideas, mastery-oriented children would probably respond better to this part of the lesson because they do not find it as threatening as do learned helpless and self-worth motivated children. Similarly, the second part of every CA lesson, *cognitive conflict*, requires children to experience or to realise that the new information is contradictory to their existing or previous theories. According to the definition of the three different motivational styles, it is again the mastery-oriented children who would be the only ones to enjoy the challenge and not to perceive it as threatening. For this reason, Leo and Galloway (1996) use the term



‘cognitive assault’ when referring to the experience of cognitive conflict in the case of learned helpless and self-worth motivated children.

Relating to the *metacognitive* part of the lesson, Leo and Galloway (1996) claim that older children would probably have a better experience because, as they grow up and mature, they also develop the ability to value learning itself and to decrease the tendency towards a more superficial approach to learning that only values rewards (peer approval, teacher recognition and good grades). In addition, promoting teachers’ awareness of their students’ metacognitive strategies could have had a positive impact on teachers’ expectations about their students, especially those who were usually classified as low achievers. In turn, children’s perceptions of the positive change their teachers and peers experienced regarding their ability could have had a positive impact on their motivational style.

The last part of the lesson, *bridging*, encourages students to apply what they have learned to other, new situations. Leo and Galloway (1996) argued that, in order to do this satisfactorily, the students need to have a certain level of control over their learning process. For this reason, they stated that only mastery-oriented children would benefit from this part of the lesson, since they are the only ones who feel that learning outcomes depend on their efforts. However, learned helpless and self-worth motivated children could also benefit from the small group work and safe classroom environment that is usually present in CA classrooms, since they would not feel as threatened as they would in a regular classroom.

Leo and Galloway’s (1996) motivational framework could be useful for understanding the differential effects that CA has produced in students. However, it is relevant to bear in mind that the link they made was exclusively theoretical or, in other words, hypothetical in the sense that they never explored their claims. In this context, the doctoral research carried out by Rosalind McLellan (2006) tried to investigate whether motivational goals or world-view theories could account for these differential effects.

McLellan (2006) assessed the motivational styles, or world-views, of 1600 UK students attending five different secondary schools by using a goal-theory approach to motivation, related beliefs, self-concepts and motivational orientation.

Five of the nine schools were implementing CASE in their secondary classrooms and were evaluated at the beginning and at the end of the CASE application. According to McLellan (2006), students' motivational style is intimately related to the type of goals they pursue. She identified the existence of two main different types of goals:

- i. *Task goals*: When students have this type of goal, their objective is to learn and, as a result, students will make their best efforts in order to succeed because they are convinced that success is the consequence of effort. This type of student is called 'task oriented' and s/he considers mistakes to be a necessary part of the learning process; therefore, such students do not feel threatened by them.
- ii. *Ego goals*: When students have this type of goal, their aim is to be considered more able than are others. Putting a lot of effort into something is seen as a sign of low ability. The tendency to adopt this kind of goal is called 'ego orientation'. Ego goals are usually divided into two categories, namely *approach* (the main purpose is to demonstrate high ability) and *avoid* (the main purpose is to avoid being seen as low ability).

Similar to the characterisation made by Leo and Galloway (1996), the impact of these types of goals is that they promote different approaches to the learning situation, with one of them (task orientation) being more productive for learning than the other (ego orientation).

In her study, McLellan (2006) found that only 13% of the students held task goals at the beginning of the CASE intervention and that 50% of them were more concerned about achievement and appearing as high achievers in front of others (teachers and peers). In addition, students participating in CASE lessons changed in a more adaptive way (they moved to more positive world-views) than did their peers in the control groups. Similarly, within the control schools, more students maintained their maladaptive goal orientation (ego goals) than in experimental schools, where more students tended to move towards more adaptive goal orientation (task goals). In other words, students who participated in CASE lessons were more concerned about learning at the end of the intervention than

were their control counterparts, who tended to maintain their concern about being seen as high ability students or not being seen as low ability students.

However, in terms of cognitive development, students who held more adaptive world-views did not make greater progress than those who held maladaptive world-views. In fact, one sixth of the students significantly improved their cognitive development even if they had had a maladaptive world-view before the beginning of the CASE intervention and they maintained that world-view at the end of it (2 years later). Also, approximately one fifth of the students held more adaptive goals and made only small cognitive gains.

McLellan's (2006) findings were not sufficiently conclusive and, for that reason, she claimed that the relationship between cognitive development and motivational goals, or world-views, is highly complex and is probably mediated by several factors such as teacher quality, classroom environment, curriculum materials and so on. In this context, even though looking at the motivational style theory is an interesting enterprise for finding new explanations for the gains many students have made by participating in cognitive acceleration programmes, the results produced so far are not sufficiently conclusive to consider motivational theory as a strong and grounded alternative explanation for CASE results. In this sense, it would be interesting to have other research evidence that could complement these findings; thus, new studies concerning this issue are required.

## **IV. Literature Review: Part II**

The first part of this literature review described the nature of formal reasoning skills and ways in which cognitive acceleration programmes have tried to develop them in school students over the last three decades. However, this research explored the experience of using the CA approach with a completely new population in a different country: prospective primary teachers in Chile. In this context, a legitimate question would be, ‘Why it is possible to think that this approach could be beneficial for the training experience that prospective teachers have during their Bachelor of Education programmes?’

As a researcher, part of my assumptions are grounded in the success that cognitive acceleration programmes have had over the past three decades and the meaningful learning experiences that they have provided for the participating school students. However, transferring the results with school students to prospective teachers (university students) is not completely straightforward; therefore, it is necessary to discuss what a good teacher needs to know and to be able to do, as well as how thinking skills play a role in effective teaching practice.

### **What does a teacher need to know?**

In this era of education reform, discussions regarding the knowledge that characterises an effective teacher and how such knowledge could be developed through training instances is of central importance. The attention given to these issues is reflected by the amount of research that discusses effective practice and the number of political and technical communications issued by governments all over the world establishing guidelines to achieve these highly valued goals (Stones, 1992; Ball and Cohen, 1996; Turner-Bisset, 1999; Jegede et al., 2000; Kennedy et al., 2008).

Focusing on what makes an effective teacher is not a new issue and, in the 1980s, Lee Shulman and his colleagues had already proposed a model for understanding effective teaching by identifying and differentiating its components

to the American Educational Research Association. One of the major contributions of this proposal was the recognition that there is a special kind of knowledge, pedagogical content knowledge, that is particular to the teaching profession, since it incorporates understanding the elements of the subject matter that are relevant for making it accessible to others.

In general terms, the model proposed by Shulman (1987a, p. 8) states that teachers' knowledge could be structured according to seven different categories:

- i. *Content knowledge* is the understanding of the subject matter itself, with its relevant principles and relations between them.
- ii. *General pedagogical knowledge* refers to the expertise needed for managing the classroom in general, its complexities, contingencies, strategies and so on.
- iii. *Curriculum knowledge* is usually related to the list of contents and abilities, materials and programmes mandated by the government for each school grade.
- iv. *pedagogical content knowledge* is described by Shulman as the special amalgam of content and pedagogy that is uniquely the province of teachers, and which is their own special form of professional understanding
- v. *Knowledge of learners and their characteristics* means that teachers should know their learners in depth in order to make classroom and teaching decisions based on that information.
- vi. *Knowledge of educational contexts* means that every classroom and school is inserted in a broader context (political, economic and cultural), which has to be taken into consideration at the time of planning.
- vii. *Knowledge of educational ends* refers to purposes and values, and their philosophical and historical basis.

I will now proceed to talk about the two kinds of knowledge that have usually been considered most important for effective teaching and student achievement, namely pedagogical content and subject matter knowledge (Howey and Grossman, 1989, Peterson et al., 1989, Barnett, 1991, Ormrod and Cole,

1996, Ball et al., 2008, Hill et al., 2008, Akkoç, 2010, Baumert et al., 2010). For further review on the others types of knowledge, see Shulman (1986) and Shulman (1987a).

#### *A. Pedagogical content knowledge (PCK)*

As stated above, pedagogical content knowledge is considered by Shulman (1987a) to be the most important category of the seven that he identified. He describes it as a particular kind of knowledge needed for teaching, since it is composed of both the knowledge of the discipline's content and the specific pedagogical practices and understanding for making it accessible to the students. In this category, Shulman (1986, p. 9) included "the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations".

As can be seen, PCK is the kind of knowledge that differentiates an effective teacher from a subject specialist, an expert, or a scientist. In this sense, "while a scientist's knowledge is structured from a research perspective and is used as a basis for the generation of new knowledge, a science programme should facilitate a science teacher's understanding of science from a teaching perspective so that it can be used as a basis for helping specifics students understand specific concepts and learn distinctions between similar concepts" (Cochran et al., 1993, p. 267).

In other words, PCK includes procedural and conceptual understanding that informs teachers' practices which, in turn, promote and encourage the development of meaningful learning in students (Kanes and Nisbet, 1996). Essentially, PCK is the practical knowledge needed for teaching any school subject, since it involves a deep understanding of the subject that allows the teacher to represent each topic in multiple ways, taking into account students' previous knowledge, conceptions, common mistakes and difficulties, tasks, activities, problems and explanations (Ernest, 1989).

For example, in the case of mathematics, PCK refers to the ability to represent certain concepts, such as negative numbers (Sternberg and Horvath, 1995), or how to prove and demonstrate certain procedures and methods (Leinhardt, 1987), and to be aware of common students' misconceptions and naïve theories in various contexts (Gardner, 1991). Similarly, explaining and calculating a circumference perimeter is very different to responding to a student's comment about the relationship between perimeter and area. The first only requires having some knowledge of the procedures, while the second involves flexibility and the ability to think about the relationship between different concepts (Ball et al., 2008).

One of the purposes put forward by Shulman (1987a) when developing this model was to attract attention to the fact that teachers were not being sufficiently prepared in terms of the knowledge and tools they needed in order to carry out their teaching task successfully. Traditional teacher education programmes, he claimed, did not focus on pedagogical content knowledge, but on subject matter. In fact, Kanes and Nisbet (1996) used an open-ended question survey in order to explore primary and secondary teachers' views on teachers' functions and student learning. A total of 44 teachers from 10 different schools in Brisbane (13 teachers in 4 primary and 31 teachers in 6 secondary schools) participated in the study, and they found that only 34% of them considered themselves to be adequately prepared in pedagogical content knowledge and 54% in subject matter.

Similar results were found in a study carried out by Jegede et al. (2000) in two higher education institutions in Hong Kong. A total of 183 in-service and pre-service science and mathematics teachers were evaluated using a 60-item instrument that intended to explore their perceptions of their actual knowledge and of what they still needed to develop in order to become expert teachers. The descriptive and inferential data analysis showed that teachers were not very confident about their knowledge. In a 5-point scale, the mean was 3.34 for procedural knowledge, 3.25 for pedagogical content knowledge, 3.24 for pedagogical knowledge and 3.17 for content knowledge.

Given the novelty and utility of the concept, many scholars began to use it (Grossman, 1987; Howey and Grossman, 1989; McDiarmid et al., 1989; Peterson et al., 1989; Wolfe and Murray, 1990; Askew et al., 1997; Ormrod and Cole, 1996) and tried to explore its empirical validity. For example, Turner-Bisset (1999) conducted a two-year longitudinal study that consisted of observing prospective primary teachers learning to teach. The research team found that Shulman's categories were not discrete and sometimes overlapped. Consequently, they redefined Shulman's model by eliminating some categories that did not seem to play an important role in the teachers' practice (knowledge of educational ends, purposes and values), and by including others such as dividing content knowledge into three smaller categories, as follows (Turner-Bisset, 1999, p. 44):

- i. Substantive structure – this consists of the facts and concepts of a discipline, and the organising frameworks used to marshal what may appear to be a profusion of disparate pieces of information.
- ii. Syntactic knowledge – this is the ways and means by which the propositional knowledge has been generated and established
- iii. Beliefs about the subject

Another reformulation of the concept of PCK was developed by Cochran et al. (1993). They argued that it is not clear whether Shulman's epistemological position was constructivist. Therefore, they proposed a new term, pedagogical content knowing (known as PCKg), in order to stress its dynamic nature in contrast to the static connotation of word knowledge. They defined PCKg as the "teacher's integrated understanding of four components of pedagogy, subject matter content, students characteristics, and the environmental context of learning" (Cochran et al., 1993, p. 266). Emphasising the dynamism of the concept implies that the knowledge every teacher should develop undergoes constant modification and restructuring in order to create appropriate teaching practices for promoting the construction of specific understanding in students in a specific context.



### *B. Subject matter or content knowledge (CK)?*

This kind of knowledge is the purest form of understanding that a teacher has of the discipline s/he is teaching. However, it requires the teacher to go further than the isolated facts and concepts, since the understanding of the particular structure that gives sense to the discipline will allow the teacher to explain not only what, but also how and why to the students (Shulman, 1986). Similarly, Prestage and Perks argued that subject matter is the “knowledge about its structure, the body of concepts, facts, skills and definitions, as well as methods of justification and proof” (2001, p. 102) that students require to pass formal examinations (tests and exams) and to find solutions to problems.

The importance of subject knowledge has been emphasised by scholars and policymakers (i.e. Kanies and Nisbet, 1996, Kahan et al., 2003) by claiming that teachers are not able to teach what they do not know. For this reason, most teacher education programmes in England and the United States have emphasised the promotion of subject knowledge mastery (Bennett, 1993; Aubrey, 1997; Goulding et al., 2002; Murphy, 2006; Burgess and Mayes, 2008; Kim et al., 2011).

This trend was officially established in a circular issued in Britain by the Council for the Accreditation of Teacher Education (CATE) that states, “students' [teachers] mastery of a subject and its application facilitates more effective teaching and promotes better learning experiences for children. Hence student teachers are expected to have subject expertise in one or more areas of the curriculum and to receive tuition in the application of their specialist subject(s) to the teaching and assessment of pupils” (McNamara, 1991, p. 114).

### *C. Criticism of the distinction between PCK and CK*

Although the initial attempt to deconstruct the knowledge that teachers should have was rapidly assimilated by many scholars, researchers and governments, others were more sceptical or, at least more cautious, about trying to

disaggregate teachers' knowledge into different categories (McEwan and Bull, 1991; McNamara, 1991; Brown et al., 1999; Turner-Bisset, 1999). For example, McEwan and Bull (1991) have criticised Shulman's differentiation of CK and PCK by stating that it has no empirical justification, since all the teacher's knowledge is somehow pedagogical. Other criticism has been related to the static nature of both constructs. Some scholars have stated that the lack of dynamism in the model implies an absolutist paradigm of the discipline and, at the same time, promotes a non-constructivist view of teaching and learning (McNamara, 1991; Stones, 1992; Cochran et al., 1993; Meredith, 1995).

Similarly, McNamara (1991) argued that, even though it is possible to create a model that differentiates CK and PCK in theory, it is not clear if it is possible to do the same in practice, because CK is already a form of representation. These critics have been supported by research findings that have shown that it is impossible to distinguish between CK and PCK in the act of teaching by observing teachers' classroom discourse (Bennett and Turner-Bisset, 1993; Turner-Bisset, 1999). For this reason, most teachers and teachers trainers agreed when Lucas (1993), in the introductory speech at the National Curriculum Council (NCC) conference on mathematics and Initial Teacher Training (ITT), claimed that "subject knowledge is inseparable from its application" (Meredith, 1995, p. 175).

Even though it is important to take these criticisms into consideration and to be aware that there are some scholars who have not assimilated the model as easily as have others, it is possible to state that many researchers support Shulman's framework and consider it to be a useful tool for understanding and exploring the role that teachers' knowledge plays in students' learning (Ernest, 1989; Cochran et al., 1993; Kanes and Nisbet, 1996; Geddis and Wood, 1997; Jegede et al., 2000; Hill et al., 2005; Ball et al., 2008; Shechtman et al., 2010; Kim et al., 2011) and, ultimately, in raising educational standards.

#### *D. The amount of mathematical knowledge that a primary teacher should have*

In the previous section, I described what many scholars and much research has shown in respect of what a teacher should know. However, this question needs to be narrowed down and focused, since this thesis is not related to teaching in general but to teaching primary mathematics in particular. Thus, the question is how much mathematical knowledge a primary teacher needs and how deep that knowledge needs to be.

Primary teachers usually work with relatively young children; aged 6 to 12 in Chile. Therefore, some people might argue that teaching primary mathematics is a relatively easy enterprise. However, this assumption is ill founded. As Ma states (1999, p. 149), “it is widely accepted that elementary mathematics is “basic”, superficial, and commonly understood. Elementary mathematics is not superficial at all, and any one who teaches it has to study it hard in order to understand it in a comprehensive way”.

Given the importance and complexity of teaching primary maths, an increasing amount of research has tried to establish a relationship between the depth of pedagogical content knowledge, students’ achievement (Rowland et al., 2000; Ball et al., 2001; Prestage and Perks, 2001; Barber and Mourshed, 2007; Shechtman et al., 2010) and teacher performance (Clotfelter et al., 2007; Hill et al., 2008) in order to find the key that characterises an effective teacher.

#### *Relationship between pedagogical content knowledge (PCK) and students’ achievement*

It might seem obvious to state that a maths teacher must have profound and expert mathematical knowledge, and the global trend in this regard has followed this assumption, emphasising the strengthening of teachers’ mathematical knowledge (Rowland et al., 2000; Ball et al., 2001; Prestage and Perks, 2001; Barber and Mourshed, 2007; Shechtman et al., 2010). Nevertheless,

the scientific community is still far from not only reaching an agreement of what this means in the case of primary teachers, but also from proving this claim with research-based evidence (Hanushek, 1986; Buddin and Zamarro, 2009). Murphy (2006, p. 230) describes this problem, claiming that “Although much evidence points to the need for teachers to understand mathematics in an in-depth, flexible and connected manner, there is no consensus on the role that knowledge of mathematics beyond the level being taught may have”.

Although some people would argue that there is no relationship between teachers’ mathematical knowledge and students’ achievement, I am inclined to agree with those who claim that the problem is related to the kinds of variables that have been usually considered to constitute mathematical knowledge (QBTE, 1985; Kanes and Nisbet, 1996; NMAP, 2008). In many studies (QBTE, 1985; Monk, 1994; Hill et al., 2005; Buddin and Zamarro, 2009), teachers’ knowledge has been measured using indirect variables, such as the number of maths courses taken or the highest degree obtained, which does not necessarily mean that a teacher is better prepared to teach maths or that s/he performs better in the classroom. The same conclusion was drawn in a study conducted by a team from King’s College London, who found that “what matters is not formal qualifications or the amount of formal subject knowledge, but the nature of the knowledge about the subject that teachers have” (Askew et al., 1997; p.93).

In this respect, the National Mathematics Advisory Panel from the US Department of Education describes the state of the art as follows:

Research on the relationship between teachers’ mathematical knowledge and students’ achievement confirms the importance of teachers’ content knowledge. It is self-evident that teachers cannot teach what they do not know. However, because most studies have relied on proxies for teachers’ mathematical knowledge (such as teacher certification or courses taken), existing research does not reveal the specific mathematical knowledge and instructional skill needed for effective teaching, especially at the elementary and middle school level. Direct assessments of teachers’ actual mathematical knowledge provide the strongest indication of a relation between teachers’ content knowledge and their students’ achievement. More precise measures are needed to specify in greater detail the relationship among elementary and middle school teachers’ mathematical knowledge, their instructional skill, and students’ learning (2008, p. xxi).

Given the recognition that proxy variables are not helpful for increasing our understanding of the aspects of a teacher's knowledge that are the most relevant for improvement, recent research efforts have been driven to define and measure new constructs. For example, Deborah Ball and her co-workers (2008) coined the term 'mathematical knowledge for teaching' (MKT). They defined it as the specific mathematical knowledge a teacher would require in the process of teaching mathematics. They emphasise the importance of the word 'teaching' in their definition, because they wanted to suggest that teaching maths is a dynamic and challenging process that involves specific demands.

In order to measure that construct and to explore the role it plays in teaching and learning mathematics, Hill et al. (2005) carried out a research study that used a linear, mixed-model methodology that assessed teachers' MKT in relation to the progress that their first- and third-grade students made in mathematics. Their findings were extremely interesting, since they showed that teachers' mathematical knowledge correlates positively with students' achievements, even with very young students and without highly complex content.

Other studies have confirmed the relationship between the teacher's knowledge and the students' achievement. For example, in a comparative study between Chinese and US teachers, Ma (1999) found that even though US teachers attended maths courses that were more complex during their training as prospective teachers, Chinese teachers demonstrated a more expert and profound understanding of the mathematical content taught in primary school. This might be related to the fact that Chinese students obtain much higher scores on international tests. Although this is clearly a correlation and not necessarily a causal relationship, this evidence reinforces the importance of training teachers in the knowledge at the level they are supposed to teach.

Similarly, a study conducted in Belize by Mullens et al. (1996) showed a significant relationship between teachers' mathematical proficiency as expressed in test scores and the students' attainment. However, that connection was only demonstrated in the case of advanced maths concepts. The authors explained this

phenomenon by saying that this occurs not only because teachers with higher scores demonstrate better understanding of the concepts, but (and more importantly) they are also more likely to present these concepts and deal with them during a regular lesson than is a teacher who does not feel confident with them. Consequently, expert teachers' students will not only learn better and more profound maths, but will also have a deeper maths learning experience.

Finally, international comparative studies have confirmed this relationship. Blömeke et al. (2011) put together the results from the Teacher Education and Development Study in Mathematics (TEDS-M) that evaluated prospective primary teachers from 15 countries in their final year of initial teacher training, and students' mathematics results from PISA and TIMSS. The results showed that prospective teachers from PISA and TIMSS high-achieving countries, such as Taiwan and Singapore, were those who also obtained the best results in TEDS-M.

Despite the significance of the results, some authors claim they are not sufficient for understanding all the aspects of the mathematical knowledge that matters for teaching. Based on the evidence provided by previous research, they explained their results by saying that “knowledgeable teachers may provide better mathematical explanations, construct better representations, better ‘hear’ students’ methods, and have a clearer understanding of the structures underlying elementary mathematics and how they connect” (Hill et al., 2005, p.401).

However, these are only hypotheses that future research should consider when analysing knowledgeable teachers' practice in order to provide a better and deeper understanding of what characterises the practice of an effective teacher in the classroom. In this sense, is not enough to confirm that there is a significant relationship between teachers' knowledge and students' performance, as how and why this link takes place in the classroom must be explained.

### *Relationship between PCK and teachers' performance*

As stated above, although some studies have concluded that there is a relevant connection between teachers' mathematical knowledge and students'

achievement, scholars are still seeking an in-depth understanding of such links by exploring the variables that mediate them (Clotfelter et al., 2007; Hill et al., 2008). According to Shechtman (2010, p. 349), “MKT [Mathematical Knowledge for Teaching] is likely to be a heavily mediated construct in regard to its effect on student learning, and we do not yet know what the mediating variables are or how they work”.

Another challenge in analysing the association between teachers’ knowledge and students’ achievement is that often what teachers know is not directly transferable to the classroom, or does not address the contextual demands of their everyday work (Ball et al., 2001). Therefore, scholars have needed to find ways to explore how teachers’ knowledge is reflected in what they do inside the classroom in order to grasp the real impact of teachers’ knowledge on their instructional practice.

A study conducted by Baumert et al. (2010) in Germany found that 39% of the variance between classes was explained by the difference in teachers’ PCK. The authors explained that teachers with significant differences in their pedagogical content knowledge mainly differed in terms of the level of cognitive challenge created by the tasks used during the lessons, the degree of curriculum coverage for the grade being taught and the strategies used for supporting students during their individual learning process.

Another study, conducted by Da Ponte and Chapman (2006), found a relationship between the teachers’ cognitive level and their classroom practice. The authors observed that teachers with lower cognitive levels tended to have practices that were more teacher-centred and to follow the objectives established by the official curriculum for each lesson rigidly. In contrast, teachers with higher cognitive levels were usually keener to ask for students’ opinions, appreciated individual differences and encouraged divergent thoughts and creativity during the lessons. Clearly, teachers’ confidence in the classroom and their willingness to enter into dialogue with their students was higher among teachers with higher cognitive levels.

In the UK, three universities (the University of York, the University of Cambridge and the Institute of Education) collaborated in a study in which the

efforts were oriented towards exploring how much and in which ways mathematical subject knowledge impacts on teaching practices at the primary level. The teachers' mathematical knowledge was measured by an audit that covered areas that were relevant to the primary curriculum. The research team found that "poor subject knowledge as identified by the audit was associated with weaknesses in planning and teaching primary mathematics (...) students with low audit scores were more likely than other students to be assessed as weak numeracy teachers" (Goulding et al., 2002, p.699). Similarly, Askew et al. (1997, p. 5) found that "highly effective teachers of numeracy had knowledge and awareness of inter-relations between the areas that they taught of the primary mathematics curriculum". In other words, teachers do not need to have a degree in mathematics to be an effective numeracy teacher in primary school, but they do need to have a degree of knowledge that is coherent and which is not compartmentalised.

Teachers' discourse is another aspect of their practice that is affected by the amount of mathematical knowledge. Khisty (2001, 2002) observed that teachers who were more knowledgeable not only presented a mathematically rich discourse, but also communicated high expectations of their students' progress, intentionally made use of questioning as a learning tool, promoted students' peer and group collaboration and were proficient at introducing mathematical concepts in diverse situations.

These results were corroborated by another study that showed the relevant role that instructional discourse plays in effective teaching (McDonough and Clarke, 2003). The authors concluded that effective teachers usually probed the level of understanding and thinking during their students' progress, were skilful at engaging students in the different tasks and were conscious of the kind of maths experience their students needed in order for growth in understanding.



### *E. The role that reasoning plays in effective teaching*

The recognition that there is a special kind of knowledge that is particular to teachers is not the only part of Shulman's theory that has gripped the attention of scholars and researchers. A year after the publication of the article concerning pedagogical content knowledge, Shulman presented a model that placed pedagogical reasoning at the very centre of effective teaching. He supported the relevance of his proposal by saying that although the literature carefully describes what we understand by effective teaching in terms of classroom management, there is a lack of description of how ideas should be dealt with through classroom discourse (Shulman, 1987b).

When developing the model, Shulman and his colleagues followed Piaget's legacy in describing the growth of knowledge from childhood to adolescence but, in Shulman's case, the observations were oriented to identify the growth of knowledge regarding the content and pedagogy in prospective teachers who were learning to teach. As a result of years of observing prospective teachers becoming experienced teachers, they concluded that teaching is essentially a process of reasoning and comprehension, since teachers need to learn how to use their knowledge base in order to make pedagogical decisions (Shulman, 1987b).

The model involves six steps that, although they are not rigid, summarise how teachers' knowledge evolves in terms of both comprehension and action in order to provide feedback for future pedagogical reasoning. The stages described by Shulman (1987b) are the following:

- i. Comprehension: in order to teach, it is crucial to understand not only the content, but also the purpose, and to position these in relation to the overall picture of the discipline
- ii. Transformation: after comprehension, the teacher needs to transform the knowledge from his or her own point of view to one that is accessible to the students or that is teachable
- iii. Instruction: this is the actual process of making the content accessible to the students and is expressed in observable actions in the classroom

- iv. Evaluation: involves both informal instances, in which teachers check their students understanding during a lesson, and more formal ones that usually take place at the end of a unit and which are graded
- v. Reflection: the teacher looks back and reviews, criticises and improves the teaching-learning process based on his/her conclusions
- vi. New comprehension: prior knowledge is expanded, based on the previous experiences.

Many scholars have agreed with Shulman's statements that pedagogical reasoning is one of the most important teaching features for promoting students' learning and that there is a lack of research regarding the training contexts that are the most appropriate for fulfilling this aim; thus, the scholars have tried to find valid answers to these questions (Barnett, 1991; Richards et al., 1995; Herman, 1998; Sánchez and Llinares, 2003). In this context, Stoiber (1991) conducted a research programme that involved three different groups, namely a control group, an experimental group that was exposed to technical training (TEC), and an experimental group that was exposed to reflective training (REF).

The two experimental conditions differed mainly in the goals of and the teaching strategies employed during the lessons. Specifically, the technical training consisted of presenting one of the eleven prescriptive principles published in the Organising and Managing the Elementary School Classroom training programme for improving classroom management. On the other hand, the reflective training condition was oriented towards creating an analytical and problematical learning context in which prospective teachers would have the opportunity to think about and discuss various strategies that could be helpful in dealing with hypothetical cases that involved a variety of classroom challenges. Finally, the control group received the same training that it would have received if it had not participated in the research project (Stoiber, 1991).

In order to evaluate the differences, the research team took pre- and post-tests and a statistical analysis of the variance between them was conducted. Based on this, they concluded that learning environments that focus on reflection and on processes that are more constructivist are more effective in promoting pedagogical reasoning and problem-solving skills. In other words, students in the reflective

training condition obtained statistically and significantly higher scores in reasoning and problem-solving tests than did their technically trained and control group counterparts (Stoiber, 1991).

Another study developed by Perterson (1995) made use of a problem-based approach in order to enhance prospective teachers pedagogical reasoning skills. The author concluded that the approach was adequate for the purpose of the study, since it gave the students the opportunity to apply their knowledge of the content and facilitated the awareness of their own learning needs. This increased their comprehension of the teaching/learning process and the role that reasoning plays in it.

More recently, research by Nilsson (2009) and Starkey (2010) has confirmed that teacher training instances that try to involve reasoning and reflection as an important ingredient not only promote pedagogical reasoning, but also contribute to the development of meaningful connections between teachers' theoretical knowledge and practice. This, in turn, helps to break the vicious circle of teaching as the transmission of information. According to Nilsson (2009, p. 239), "The apprenticeship of observation combined with the fact that teaching as telling tends to prevail in their teacher education classes means that, for many student teachers, coming to understand teaching as being problematic and therefore moving beyond expectations of learning to teach as being 'told how to do teaching' is a constant challenge".

In summary, it has been possible throughout this chapter to observe that teachers' effectiveness and their ability to promote students' meaningful learning depends on a vast range of knowledge, including content knowledge and pedagogical content knowledge, as well as teaching skills, such as reasoning skills and discourse. Evidently, the topics covered during this section are not sufficient for describing all the factors and variables that play a role in teachers' practice. In this sense, it is clear that teaching is an extremely complex enterprise that cannot be reduced to a checklist of observed behaviours. Thus, the intention of this chapter was to review the topics concerning teachers' effectiveness that are directly related to this research in order to have a better understanding of the variables that were considered when conducting this study.

In other words, the purpose of writing a section that describes what primary teachers need to know in order to be effective and to promote students' learning is to make the link with cognitive acceleration programmes and the role they could play in enhancing the learning experience and the preparation to teach that prospective teachers have when finishing their Bachelors of Education degrees. However, understanding the particular educational situation of Chile as the context of this research will provide further information regarding the reasons that the CA approach could be beneficial for prospective Chilean teachers and will make explicit my assumptions and motivations as a researcher for improving the current situation in Chile.

## V. Methodology

### Preliminary considerations

This study explored how prospective primary teachers in Chile responded to a course that used the Cognitive Acceleration approach in the context of learning mathematics. In other words, the main research question I am trying to answer is, ‘how do prospective primary teachers perceive a cognitive acceleration approach in relation to teaching and learning mathematics?’ In order to address this research question, the methodology I used was one that was coherent with the interpretative paradigm.

Before going further into the particularities of this research project, it is necessary to clarify the difference between the terms ‘methodology’ and ‘methods’, because they are often used as synonyms (Dillon and Wals, 2006), which could dull the understanding of the chapter. Although the terms are interrelated, they refer to quite different aspects of research. On one hand, methodology is the set of philosophical assumptions that a researcher holds in relation to the world (ontology) and how one accesses it or, in other words, how we generate knowledge (epistemology). On the other hand, methods are the specific techniques or tools that we use for conducting research and they are, to some extent, the consequence of our philosophical assumptions. In this sense, the kinds of beliefs and claims a researcher makes about the world and about knowledge will guide not only the kind of research tools he or she uses, but also the kind of questions he or she asks.

The different methodological positions in education, with their assumptions about the world and knowledge, have traditionally been organised according to three different paradigms, namely positivistic, interpretative and critical (Husén, 1988, Candy, 1989, Cohen et al., 2007). According to Kinash, “A *paradigm* is a matrix of beliefs and perceptions. There are power relationships and action implications inherent in paradigms” (2007, p.1). Each paradigm has its own particularities and tries to differentiate itself from others based not only on the

kinds of assumptions it makes, but also in terms of the impact it expects its research to have. It is beyond the scope of this work to describe these different paradigms in detail, but it is important to define and differentiate the terms in order for this chapter to be clear and straightforward. In addition, every researcher is expected to reflect on his or her philosophical assumptions and to make them explicit at the time of reporting the research results, because there are different ways of judging research quality within each paradigm. McGregor and Murnane (2010, p.419) stated that “trustworthiness and diversity of [research] depends on researchers accounting for the methodological (philosophical) underpinnings of their work, not just the methods used to sample, collect and analyze data and report the results”.

It is also important to note that, in this work, the terms ‘qualitative’ and ‘quantitative’ research will be used as two different kinds of methods, and not as methodologies. Although it is very common to find allusions in the literature to the terms quantitative and qualitative as two different paradigms (Rist, 1977, Bryman, 1984), I tend to agree with those (i.e. Guba and Lincoln, 1994, Ponterotto, 2005, McGregor and Murnane, 2010) who state that they are a group of two different research methods (interviews, case studies, surveys, questionnaires) that should be used to deal with research questions that originate in any paradigm. In that sense, even though certain quantitative and qualitative methods are more frequently used in specific types of research methodologies, they are tools in the service of research and do not inherently belong to any particular field. According to Guba and Lincoln (1994), “both qualitative and quantitative methods may be used appropriately with any research paradigm. Questions of method are secondary to questions of paradigm, which we define as the basic belief system or worldview that guides the investigator, not only in choices of method but in ontologically and epistemologically fundamental ways”.

McGregor and Murnane (2010) stated that the differences between paradigms can be organised around three topics, as follows:

- i. *Epistemology*: what is understood by knowledge and how is it accessed?
- ii. *Ontology*: how is reality defined?

- iii. *Logic*: how is the quality of knowledge judged and how are the conclusions determined?

Having said this, I will now make explicit my philosophical assumptions and those that underlie this research project. In so doing, I will start by making explicit my own philosophical assumptions and will then present the specific methods I used during this research process for selecting my sample, and for collecting and analysing my data.

### **What is behind this research project? The interpretive paradigm**

It is not easy to present my philosophical assumptions because, to some extent, this implies that I have an absolutely clear and unchanging position regarding them. The truth is that I consider this process to be evolving continuously, as my beliefs have changed during the course of this research and my research questions have developed accordingly. This does not mean that the essence of my questions has changed, but that the focus and the ways in which I planned to deal with them has altered. Therefore, in this chapter, I plan to address and to make explicit the beliefs and assumptions that directed the development of this research.

In contrast to the positivistic paradigm, which maintains that we have direct access to the world as it really is, the interpretive paradigm states that our senses mediate the process of knowing and that, within that process, social agreement is the key to constructing shared realities (Smith and Heshusius, 1986). In other words, from the interpretative point of view, the world is not ‘out there’ waiting to be discovered or explored; instead, reality has to be constructed by the researcher. Thus, in terms of the interpretive paradigm, social agreement is essential; otherwise, there would be thousands of different realities and it would not be possible to reach a consensus or to create scientific knowledge (Husén, 1988).

There are also other differences between the positivistic and the interpretative paradigms, because the positivistic paradigm states that the facts are

accessible by the researcher, while the interpretative paradigm claims that knowledge has to be constructed. This construction process is inherently shaped not only by the researcher, but also by the participants under investigation. In this sense, the researcher does not try to isolate him- or herself or the participants as factors influencing the results in the interpretive paradigm. On the contrary, the researchers value their contributions as a unique source for understanding the social and personal spheres of the phenomena being studied. According to Garrick, “The basis of their knowledge claims rests upon assumptions that make use of participants’ stories, their language, descriptions and metaphors to highlight what is important to them, i.e. the subjects of the investigation” (1999, p.148).

As a result, interpretative theorists deny the positivistic claim that the world is governed by cause and effect relations and by laws that can be identified, described and predicted. In contrast, they argue that the only way of understanding the social world, in which the rules are constructed through a process of agreement and social consensus, is from the perspective of the subjects involved in it and by accessing their reasons, rationales and intentions when acting (Candy, 1989). Blumer (1962) describes this as “doing research with people rather than on people”. As can be expected, if in-depth understanding is the target of the interpretative paradigm, the exploration of the process through which people construct meaning is of the first importance. According to Carr and Kemmis (1983, p.88), “To identify these motives and intentions correctly is to grasp the 'subjective meaning' the action has to the actor”.

Blumer (1962) calls this process of social interdependency whereby people create collective meaning ‘symbolic interaction’. This means that social interaction does not consist of people reacting to each other’s actions, but of people making meaning or rejecting others’ actions. In this context, what an interpretive researcher looks for is ways to understand and explain the process whereby people construct meaning and how the process of developing and making use of rules is influenced and shaped by those meanings (Candy, 1989).

According to Candy (1989), three concepts constitute the essence of the interpretive paradigm. These are



- i. Inter-subjectivity, which refers to the norms that are socially agreed upon and which determine what is true or acceptable in every social context
- ii. Motives, which are all the factors that cause any action or circumstance
- iii. Reasons, which are unmet expectations that influence potential actions.

These three concepts guide the work of interpretive researchers towards questions that attempt to address the motives, intentions, values, attitudes and beliefs that are behind people's actions.

In summary, Candy (1989) describes five assumptions that are usually shared by interpretive researchers:

- i. the belief that any event or action is explicable in terms of multiple, interacting factors, events and processes, and that 'causes' and 'effects' are mutually interdependent
- ii. an acceptance of the extreme difficulty of attaining complete objectivity, particularly when observing human subjects who construe or make sense of events based on their individual systems of meaning
- iii. the view that the aim of inquiry is to develop an understanding of individual cases, rather than to establish universal laws or generalisations
- iv. the assumption that the world is made up of tangible and intangible multifaceted realities, and that these are best studied as a unified whole, rather than being fragmented into dependent and independent variables. In other words, context makes a difference
- v. a recognition that inquiry is always value-laden, and that such values inevitably influence the framing, bounding and focusing of research problems (Candy, 1989p. 4).

## **The interpretive contribution to this research: my approach**

The previous section established the theoretical and practical claims that the interpretive paradigm makes for conducting research, which has its own particularities and characteristics that differentiate it from other paradigms. However, the intention of the present section is to show how the interpretative paradigm has shaped and illuminated this particular research.

Given the recognition that reality is not accessed but is constructed through a process of social interaction and consensus, the role that I played as a researcher was equally important as the one my participants played in the research project. I did not try to isolate myself from the context. On the contrary, I was in central to the context by interacting, exploring and making contributions. As a result, my perceptions were not more important than those of the participants, and neither were their perceptions were more important than were mine. Both perceptions were equally significant.

This interdependency between my role as a researcher and my participation in the research context inevitably posed a challenge in terms of my interpretation of what was happening for a variety of reasons. In the first place, not only did my position as a researcher have a specific influence on the circumstances around me, but my personal characteristics also had a certain impact, and it is not possible to separate one from the other. It was crucial to be aware of this during the entire research process, not in order to try to diminish this phenomenon, but so as to be reflective and critical about it.

Secondly, at the beginning, the relationship between the participants and me was not symmetrical, because they were the students (or the trainees) and I was the teacher (or the trainer). For this reason, it was challenging to try to establish a dividing line between the training and the research situation. One factor that helped me to do so was the recognition that the training context was a formal requisite for them in order to qualify as primary teachers, and the research context was informal and voluntary; therefore, they could leave at any time without experiencing any kind of harmful consequence. Based on that difference,

I tried to establish a symmetric, horizontal and honest relationship with the participants.

The recognition that reality and meaning are socially constructed and are not a given not only located me within the research context, but also dictated that the methods chosen for the data collection process had a variety of forms and focuses in order to maximise their explanatory potential. According to DeWalt and DeWalt (2002), a researcher can increase the validity of a research project by using a range of data collection methods, such as observations, interviews, document analysis, surveys, questionnaires or other, more quantitative methods. In this sense, by using different kinds of methods (participant observation, interviews, field notes and so on), I was acting in accord with my belief that reality is extremely complex and that such complexity could be better approached from various points of view and by using different research techniques. These research methods will be described in the following section.

## **Methods**

### *A. The purpose of the study*

As I described at the beginning of this thesis, the main research question addressed by this work is the ways in which prospective primary teachers perceive a cognitive acceleration approach in relation to teaching and learning. In other words, I was trying to explore the perceptions that prospective teachers had regarding their experience of participating in a training course that took the cognitive acceleration approach. It is important to clarify that this training experience was not explicitly oriented to train them to be CA tutors in the future, but was rather a maths course that used this approach and which was one of the optional courses offered by their Departments of Education.

Other specific research questions that this research aimed to explore were:

- i. How do the formal reasoning skills of prospective teachers change after experiencing a course on the cognitive acceleration approach?

- ii. How do the views of prospective teachers towards teaching and learning change following a cognitive acceleration course?
- iii. How do the attitudes of prospective teachers develop following a course that uses cognitive acceleration approach?

These research questions were set in relation to the following research objectives:

- i. to explore how prospective teachers view a cognitive acceleration approach in terms of experiencing the activities as learners
- ii. to explore how prospective teachers view a cognitive acceleration approach in terms of the application of the activities to teaching and learning mathematics
- iii. to explore how prospective teachers views regarding the importance of thinking change following a cognitive acceleration course
- iv. to explore how prospective teachers view a cognitive acceleration approach in terms of their confidence in teaching mathematics.

Although I believe that cognitive acceleration programmes have been successful in achieving the goals they pursue by developing formal reasoning skills, the main focus of this research project was not exclusively oriented towards the development of reasoning skills measured by a test, but to explore the learning experience that prospective primary teachers would have in a course that used the cognitive acceleration approach. There were two reasons that I was not sure whether prospective teachers' improvements would be reflected in the Science Reasoning Task (SRT) test (the test that has been used in CA research to evaluate the impact on learners). The first is related to the timeframe of a PhD research project. Cognitive acceleration programmes have been designed and implemented as interventions that are carried out during two school years. This is because cognitive development takes time.

The second reason is that CA programmes have been used in English-speaking countries (primarily Great Britain) with school students. In this project, I used this approach not only with a completely different population (prospective primary teachers), but also with a different language group (Spanish, Chile) with all the cultural and contextual factors that this implies. Although I adapted the

activities to the new context and had the chance to pilot some of them one year before the beginning of the main study, it is not straightforward to assume that the results that the approach has had in England with school students will be the same in Chile with prospective primary teachers. Having said that, I am also deeply convinced that the approach itself is a meaningful learning experience that can illuminate the training stage of Chilean primary teachers (see Chapter III, p.81).

Even though I was aware of the limitations of time and language, I was still hopeful, since CA programmes and the Science Reasoning Tasks have been implemented in a variety of different countries and no major differences attributable to cultural factors have been found (Rogan and MacDonald, 1983, Mohapatra and Mahapatra, 1998, Iqbal and Shayer, 2000, Mbano, 2003, Prophet and Vlaardingerbroek, 2003, Endler and Bond, 2008, McCormack et al., 2010).

Therefore, I committed myself to the exploration of the experiences prospective teachers would have during a course that took a cognitive acceleration approach.

### *B. The mixed method approach*

Based on all the assumptions and beliefs I have already described, I decided to take a mixed method approach in this research. A mixed method approach refers to a range of different situations, as it could mean the combination both qualitative and quantitative methods, two different kinds of qualitative methods (such as in depth interviews and participant observation) or two different kinds of quantitative methods, such as surveys, within the same research project (Hesse-Biber, 2010). In this research, the term ‘mixed’ will refer to the first two examples. In other words, I will not only use three different types of qualitative methods (interviews, learning journals and field notes), but also quantitative ones like the formal reasoning skills test (SRT).

However, it is important to clarify that the quantitative data will only provide evidence for one of the research questions, the one that deals with the effect of the CAME program on prospective teachers formal reasoning skills. As a

consequence, only that question will be approached from a quantitative point of view based on the SRT results and from a qualitative point of view based on prospective teachers own perceptions about the improvements in their ability to think formally. In turn, all other research questions this study is exploring will be approached from a mix of only qualitative methods (interviews, field notes, and learning journals).

Although some authors (Johnson and Onwuegbuzie, 2004, Hesse-Biber, 2010) have argued that using qualitative and quantitative methods together is a powerful combination because the first adds meaning to the second and means that the results can be generalised for subsequent scrutiny, this is not the rationale behind using a combination of methods in this research. As I claimed in the previous section, I am deeply convinced that reality is highly complex and multifaceted and, therefore, a good way of approaching it without losing its inherent meaning is by using multiple methods that are oriented to different aspects or points of view of that complexity. In addition, by using mixed methods, I am increasing the validity and reliability of my results, which is an added value.

With regard to mixed methods research, Husén (1988) argued that “many, perhaps most, problems in education can certainly be better investigated when examined by means of different approaches” . However, research that uses mixed methods in educational research is uncommon. Gorard (2002) states that, in a presentation made by Hausman to the annual American Educational Research Association conference in 2000, he pointed out that qualitative research was three times more common than was using quantitative research, and that research using both methods was the least frequent. This paucity of mixed method research is a pity because it has many advantages, such as making it possible to relate and compare different evidence, especially when conclusions are not conclusive or are conflicting, and it allows one to explore different facets of the same phenomena, as well as having the potential to be used as a way of triangulating information from different methods (Creswell, 2009).

### *C. Pilot study*

#### *Objectives*

A pilot study was carried out in Santiago, Chile, in August 2011 with the purpose of trialling a group of cognitive acceleration activities in order to evaluate their adequacy and usefulness for working with prospective Chilean primary teachers. Since CAME activities have not previously been used with students older than 16 years of age, it was important to explore if this was possible.

I decided to trial only five activities (see Appendix A and B) for two main reasons. The first was practicality, because I had to travel from London to Chile to conduct the CAME sessions. Thus, I faced time restrictions as I could not stay in Chile for an extended period. On the other hand, five seemed to be a sufficiently large number because, for the main study, I planned to use 16 activities. In this context, five represented almost 30% of the activities of the main study.

The pilot study also intended to explore if it were possible to use a paper and pencil test for assessing prospective teachers' formal reasoning skills.

#### *Adapting CAME activities*

For the pilot, I adapted some of the CAME activities that were to be used for this study. The selection of the activities to be trialled was guided by the aim of covering as many reasoning patterns and reasoning levels as possible, in order to have a better picture of prospective teachers' responses to a variety of them. Although CAME sessions are not exclusively organised in terms of complexity, they do follow an order in the sense that later activities usually start from a point that includes some reasoning patterns that were developed during previous sessions. As a result, I selected the activities with this in mind. For details regarding the activities selected for the pilot study, please refer to Appendices A and B.

Once I had selected the activities that I was going to trial, the next phase consisted of adapting them to the appropriate context. One of the most important features of each cognitive acceleration session is that they generate the necessary motivation on the part of the participants. Thus, the participants are engaged with the activities in order to encourage participation, discussion and dialogue with others, which is a central aspect of the suitable development of each lesson. For this reason, it was important to bring the activities closer to prospective teachers' worlds and interests. Therefore, I changed the perspective of each activity from one in which prospective teachers were treated as students developing some math problems to one in which I introduced a problem that could occur in their future professional practice as teachers.

To illustrate this point, I will present the original headline of Activity 20, 'Heads or tails', and the changes I made to it in order to make it more suitable. Initially, the instructions for Notesheet 1 read:

*Spin or throw your coin each time.*

*Write H if it comes down heads, T if it comes down tails.*

*For each group of 10 tosses, write down how many heads came down. We take each group of 10 throws as a sample.*

*Complete all 50 throws before answering the questions below.*

The activity is very interesting for developing the concepts of probability and chance, and is similar to the experience that future teachers had as schools students. However, I wanted to create a hypothetical context or problem that prospective teachers could face in their professional experience. Thus, I rephrased the problem as follows:

*Next week, your school will be running a football championship and each class has to present two different teams. Your students have decided they prefer to be randomly allocated into one of them. Therefore, you proposed that each student throws a coin. One team will be 'heads' and the other 'tails'. However, some students complained, since they think that if the person before them gets 'heads',*



*they will probably get 'tails' and thus do not have the same possibility of throwing heads or tails. Do you agree with them? To find out who is right, develop the following experiment.*

As can be seen in both the original CAME activity and in my adaptation for prospective teachers, the problem and its solution are the same but, in the second format, the emphasis is on the students' roles as future teachers and not on school students. Therefore, the situation encourages them to think hypothetically and to try out different solutions or perspectives for solving a problem that could occur in their role as teachers, and which is not just a theoretical mathematical problem they have often encountered during their experience as students.

Each pilot session lasted between 50 and 60 minutes and I followed the same structure during all of them. At the beginning of each activity, I presented the students with the problem that we were going to solve during the session. They were then to work in pairs or in groups of three persons, depending on the number of students who attended the session, in order to try to find a solution. As discussion and reflection are key components of every cognitive acceleration activity, I usually asked them to work with someone who did not think the same way as they did. For example, during the 'Heads or tails' activity described above, the headline said, *'Do you agree with your students? To find out who is right, develop the following experiment'*. Consequently, a student who agreed with the statement *'Each student has the same probability of throwing heads and tails'* had to work with another student who disagreed with this statement.

When most of the groups had found a solution, we developed a class discussion in order to share ideas and to analyse if there was a solution procedure that seemed to be better than were others and in which contexts the other solutions could be more useful. At the end of the session, the students had to think about future applications of the skills the activities attempted to develop. For more details regarding the structure of cognitive acceleration activities, please refer to Chapter III, p. 56.

### *Measuring formal reasoning*

After adapting the activities, it was necessary to deal with the second objective, namely to explore if it were possible to evaluate students' formal reasoning patterns using a paper and pencil test. There is a wide range of tests that pursue this purpose (Rowell and Hoffman, 1975, Lawson and Blake, 1976, Carlson et al., 1977, Lawson, 1978, Tobin and Capie, 1981, Shayer et al., 1981, Arlin, 1982, Roberge and Flexer, 1982) and it is therefore possible to find various formats and many arguments in favour of or against them.

In this context, the most relevant aspect to keep in mind was that choosing a test is always an option that entails advantages and drawbacks. Accordingly, it was necessary to make a decision by trying to find a test that would not only fulfil my research purposes, but which would also have acceptable validity. Therefore two of the tests seemed to be more appropriate, the Operational Thinking test developed by Barbro M. Bergling in Sweden (Bergling, 1990) and the Science Reasoning Tasks designed by Michael Shayer and his colleagues in the United Kingdom (Shayer, 1977).

Bergling's test was suitable because it is a relatively easy test to administer to large groups of people. It is not only a paper and pencil test, but also has a multiple-choice format, so I did not need any special equipment or have to perform real time experiments in order to administer it. In addition, the test is validated by the International Association for the Evaluation of Educational Achievement (IEA) and it was used in Sweden as a national option for the Second International Science Study (SISS). The disadvantage of using Bergling's test was that it was designed and always used to assess younger school students; therefore, the scoring method was not directly transferable to the new context.

In turn, the Science Reasoning Task test (SRT) was easier to mark because the rules were clearly detailed in the manual thereof, the scoring process did not vary from one context to another, and it had previously been used in different contexts (Rogan and MacDonald, 1983, Prophet and Vlaardingerbroek, 2003, Mohapatra and Mahapatra, 1998, McCormack et al., 2010). At the same time, each score was transformed into a thinking stage equivalence that gave me much

more information regarding the evaluated person's reasoning skill level. In addition, these are tasks that cognitive acceleration research has been using for a long time in order to assess the programme's impact (Adey and Shayer, 1990, Adey and Shayer, 1994, Shayer, 1996, Adey et al., 2002, Adey and Shayer, 2002).

On the other hand, although SRTs are considered to be paper and pencil tests, they required the performance of experiments about which the evaluator has to ask certain questions. Each student has his or her own Notesheet on which to record responses. This means that the size of the group being assessed on a single occasion is limited, since everyone needs to see what is happening during the experiments clearly. Also, the tests need to be demonstrated by someone competent and confident in performing the experiments. For these reasons, SRT tests seemed to provide limitations to application, although there were possibilities for overcoming these problems. For example, the experiments could be videotaped and made available to candidates on a computer or projected onto a screen in the test room. A further alternative is to describe the experiments and the results.

As can be seen, it was not possible to apply either test in its original format. Bergling's test needed new scoring rules and new ways of translating different ranges of scores into thinking stage categories. In turn, the Science Reasoning Tasks required adaptation to a new format that either does not involve experiment performances, which may affect students' attainment since they are not given the possibility of observing the processes each task is evaluating, but have to imagine them instead. Considering these factors, I decided to adapt both tests and to administer task II: Volume and Heaviness (SRT) and Bergling's test for my pilot sample, as this would allow me to compare both tests results and to decide which test would better suit my main research purposes. Thus, I eventually applied a test that included the questions from the Volume and Heaviness task (Task II, SRT) and questions from Bergling's test (see Appendix C).

From the seven existing reasoning tasks, I chose task II: Volume and Heaviness for two main reasons. Firstly, it covers the entire range of thinking stages described by Piaget; thus, it allows for the classification of the evaluated person from the early concrete stage (1) to the early formal operational stage (3A)

(see Table 11). In addition, the Volume and Heaviness task has been widely used for the purpose of evaluating formal reasoning skills in different age samples (Shayer et al., 1976, Shayer and Wylam, 1978, Howe and Shayer, 1981, Rogan and MacDonald, 1983, Hautamäki, 1986, Lim, 1988, Kutnick and Thomas, 1990, Lim, 1994, Sprod, 1998, Maume and Matthews, 2000, Budiman et al., 2009); therefore, it not only seemed more valid, but would also permit me to compare my results with those obtained in previous research.

**Table 11: Reasoning level classification**

Classification	Reasoning level
2A	Early concrete
2A/2B	Concrete
2B	Mature concrete
2B/3A	Early formal
3A	Mature formal

(Shayer, 1977)

### *The sample*

Four Departments of Education agreed to participate in the pilot study. For reasons of confidentiality, they will be called UA, UB, UC and UD. Although four Universities were initially willing to participate in the pilot study, there was a major educational strike in the country when I went to Chile to carry out the CAME sessions. Many secondary and high school pupils and university students were complaining about funding, opportunities and equity, among other issues, and they were not attending their lessons. Of the four Universities, two of them (UC and UD) were adhering to the strike; thus, it was not possible to develop the pilot at those universities. The disadvantage of this was not only that I finally worked with only two Universities (UA and UB), but also that the two participating universities were relatively similar to each other (see Table 20, p.137).

**Table 12: CA sessions schedule**

Session n°	Activity n°	Activity title	University	n	Students year
1	2	Tournaments	UA	18	1
2	15	Circle functions	UA	34	1
3	20	Heads or Tails?	UA	27	1
4	22	Comparing correlations	UA	33	1
5	30	How do I handle the data?	UA	29	1
6	22	Comparing correlations	UB	10	1
7	30	How do I handle the data?	UB	11	2

After recruiting the Universities, I asked them if they could give me access to the mailing list of my potential participants; in other words, first- or second-year students enrolled in their Bachelor of Education programme. UA had a total of 63 potential participants and they authorised me to carry out five cognitive acceleration (CA) sessions and to work with first year students only. In turn, UB had a total of 30 potential participants, 15 in their first year and 15 in their second year, and they allowed me to develop two CA sessions, one with first-year and one with second-year students. Once I had the students' contact details, I sent an email inviting them to participate in the study. Table 12 presents the number of participants of each session (n) and in which University the session was held.

### *Observations*

During the pilot study, I made detailed field notes immediately after each session in an attempt to capture facets of the experience. In general terms, based on my observations during the pilot and from the field notes I wrote, it is possible to state that cognitive acceleration activities are suitable for working with prospective primary teachers for several reasons. In the first place, prospective teachers engaged with the activities and were motivated to find a solution to each of the problems presented to them during the sessions. Secondly, although the

activities were originally designed for working with school students at KS3 and S1/S2 level, the activities adapted to the new level and context were sufficiently difficult to challenge university students' thinking. In other words, they were neither too easy nor too obvious, but had an appropriate level of complexity. Thirdly, the activities promoted the necessary reflection and group discussion in order to share ideas regarding the best way of solving each situation. Finally, student teachers' responses, questions and comments supported the hypothesis that they had not yet developed some of the reasoning patterns the activities were trying to foster, such as proportionality.

As an example, I will present some of my field notes regarding that reasoning pattern. During the *How do I handle the data?* sessions, students were supposed to interpret three different sets of data. One of them included two tables that represented the length of time a chemical reaction between two chemicals lasted when the chemicals were in different proportions and at different temperatures. Part of the instructions read:

*...when you mix the same amounts of the two chemicals. Here are the data for different temperatures:*

Temperature (°C)	20	30	40	50	60	70
Time in seconds	230	115	58	28	14.5	7

*...when you double the amount of the first chemical, keeping the amount of the second the same. Here are the data for this experiment:*

Temperature (°C)	20	30	40	50	60	70
Time in seconds	116	59	29	15	6.5	3.5

Although students were able to represent the data appropriately using a line graph and to conclude that, when the amount of the first chemical is doubled, the chemical reaction is faster, they could not see the pattern behind it – in other words, it was approximately twice as fast. Similarly, during the *Heads or tails?* activity, students were to compare the number of times they got 1, 2, 3, 4 or 5 heads in a row. It was clear that when the number of heads in a row increased the number of times they got it decreased but, again, they could not describe the pattern.

Based on my reflections and on the field notes I wrote about the activities as a group, it is possible to conclude that CAME activities are appropriate for working with prospective primary teachers, because they are sufficiently challenging, motivating and promote adequate reflections and discussion during the sessions. However, each session had its own particularities or, in other words, positive and negative aspects that I would need to take into account at the time of evaluation if I were to use them in the main study. This allowed me to make decisions regarding whether to keep, eliminate or to adapt each activity further for the next stage of the investigation.

Therefore, the next section will explain the adaptations I initially made to the CAME activities and the reasons for these decisions. I will also explain how I reflected on and evaluated each activity.

### *Session field notes and data analysis*

*Session 1-Tournaments:* The main idea of this activity was to use the context of school sports tournaments as a scenario for exploring algebraic symbols and expressions (Adhami et al., 1998). Although the lesson was interesting and the students were motivated, I realised that it did not produce the necessary challenges to promote students' thinking skills. Nonetheless, it was worthwhile implementing it as an explorative session, because I did not have any previous information about the reasoning level of the students with whom I was going to work. Although the activity was not difficult enough for the cohort with which I

was working with and I could therefore not determine the level of the participants, I at least knew that it was not as low as the level of the activity. For this reason, I decided not to use it again in the main CAME intervention programme.

*Session 2-Circle functions:* This activity used the problem of calculating the measurements of a new football field in order to promote thinking about the circumference and the area as a linear and quadratic relationship of the radius (Adhami et al., 1998). Although this activity was more suitable in terms of complexity, it had the disadvantage that its solution was closely related to mathematical procedures that the prospective teachers were likely to have encountered during their schooling. For this reason, it was difficult to generate cognitive conflict, since students generally applied previously memorised math contents, without leaving space for creative or new ways of thinking. In addition, this was a problem for the role I played during the session, because I realised that, in order to conduct a CAME activity and for fulfilling its potential, it is necessary to have strong mathematical knowledge, not only of the content, but also regarding the pedagogy thereof. As I do not have a mathematical background, I did not feel confident in listening and motivating the learners to discuss their math concepts when the activities were too focused on math content rather than on more general thinking skills. In other words, although I understood the activity and the way in which it is supposed to be solved, I lack the in-depth expertise to produce the expected effect on students. However, I did not completely disregard the possibility of using this activity again in a future CA programme, as long as I could find a proper way of adapting it. I also decided to try to seek further training in administering CAME activities by asking experienced CA trainers if I could observe them working with groups in the classroom on my return to the UK.

*Session 3-Heads or tails?* During this lesson, students had to develop an experiment in order to evaluate if they had the same chance of getting heads or tails when throwing a coin. The activity was found to be very attractive and engaging for the participants, but I experienced similar problems to the ones in session 2. The final part of the activity consisted of analysing a graph that



represented the differences, in terms of frequency, of times that the entire class got 1 head, 2 heads, 3 heads...10 heads in a row. Obviously, the number of times that the class got x number of heads in a row decreased, while the number of heads in a row increased. In order to understand this finding, it was necessary to talk about the method of calculating the probability of getting a successful event several times in a row so, once again, we were discussing mathematical concepts and procedures. The conclusion is the same as the one drawn from the previous session, in that I felt I needed more training in facilitating the CAME learning experience for it to be an effective activity with these students. However, with sufficient training, I would definitely use this activity again.

*Sessions 4 and 6-Comparing correlations.* The central purpose of this activity was to understand the relationship between two variables, expressed in three different graphs, and to find out which one had the strongest relation without using the statistical concept of correlation. I can say that the activity was highly successful in accomplishing its objectives. The problem presented was extremely challenging; thus, students shared different ideas regarding the best way to deal with it which, in turn, promoted interesting discussions. I also felt very confident during the session, because my role was more similar to that of a mediator than that of a teacher; thus, I merely encouraged students to participate, to reflect and to examine their different points of view. I will definitely use this activity in the future, albeit with some minor changes such as selecting variables that do not cause problems in the students, because they spent too much time discussing which measures were more valid for assessing students' performance, rather than concentrating on analysing which graph had the strongest relationship between the two variables.

*Sessions 5 and 7-How do I handle the data?* During this activity, students needed to find a way of organising the data of three different data sets in order to understand and interpret what was happening. The development of the session was very similar to the one observed in 'Comparing correlations', because students needed to deal with a problem that involved the use of analytical skills

without referring to mathematical concepts or procedures, such as range, average, mode and so on. I found that the activity was very helpful for generating challenge, not only because it was sufficiently complex, but also because students had to find new ways of understanding data that were not possible to analyse by following common procedures. They had to make decisions and justify them, since there was not a unique or ‘right’ way of solving it. I will include this activity in the cognitive acceleration intervention. For more details regarding what occurred in each session, please refer the relevant field notes (Appendix D).

### ***Reasoning skills test***

#### ***Results***

A total of 55 students took the reasoning skills test. The first 10 questions (1-9b) were from the Science Reasoning Task (SRT) test and the last 11 (10-20) were from Bergling’s test. The following table shows students’ performance for each question (see Table 13).

**Table 13: Students' performance for each question**

Question	Students that answered correctly	%
1	0	0 <sup>1</sup>
2	9	16.4
3	17	30.9
4	34	61.8
5	37	67.3
6	21	38.3
7	24	43.6
8	7	12.7
9a	36	65.5
9b	5	9.1
10	17	30.9
11	26	47.3
12	49	89.1
13	16	29.1
14	10	18.2
15	4	7.3
16	32	58.2
17	21	38.2
18	11	20
19	29	52.7
20	15	27.3

While Bergling's test only permits evaluation of the number of students that answered each question correctly, the results obtained in the Volume and Heaviness task (SRT) allow students' classification according to the five different

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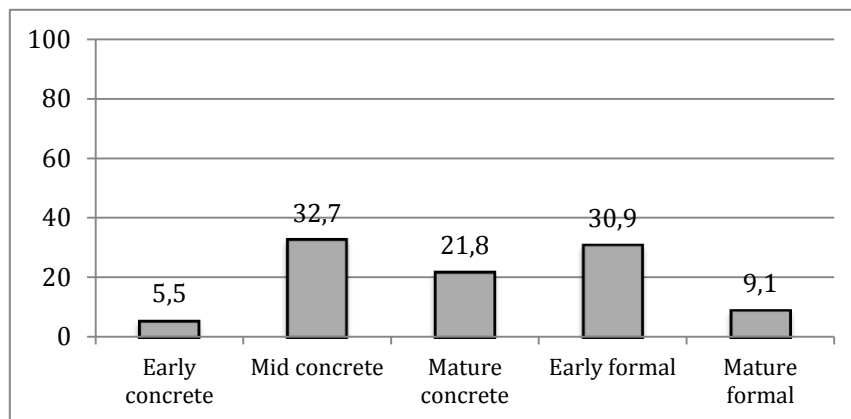
<sup>1</sup> This question is not considered in the scoring protocol for calculating the final score.

thinking stages described by Piaget, namely early concrete, mid concrete, mature concrete, early formal and mature formal. Table 14 and Figure 10 present the number of students classified according to each category.

**Table 14: Thinking stage distribution**

Thinking stage	Symbolisation	n	%
Early concrete	2A	3	5,5
Mid concrete	2A/2B	18	32,7
Mature concrete	2B	12	21,8
Early formal	2B/3A	17	30,9
Mature formal	3A	5	9,1

**Figure 10: Thinking stage percentages**



As can be concluded based on the table and the graph provided, the Task II from the SRT test indicates that only 40% of prospective teachers have already developed what Piaget and Inhelder (1958) called ‘formal thinking’ skills. This is of concern, because these skills are necessary not only for dealing with future professional challenges as teachers, but also for promoting these kinds of skills in their future students.

### *Tests' reliability and validity*

The results obtained seemed to support the hypothesis I had before administering the test; in other words, that many prospective primary teachers have not yet developed formal reasoning skills. However, reflecting that it is possible that the results obtained are only a consequence of the test(s) used is always a central part of the research process.

Therefore, the concepts of reliability and validity are frequently utilised as a measure of the quality of the tests administered. Both terms have slightly different meanings and ways of ensuring them in qualitative and quantitative research (Cohen et al., 2007). Thus, I will briefly present my understanding of reliability and validity in a mixed methods research context.

Reliability is usually understood as stability, in the sense that the data produced by the test can be obtained again if similar samples and contexts are used in the future. In this sense, as Cohen et al. (2007, p. 133) state, “a reliable instrument for a piece of research will yield similar data from similar respondents over time”. In turn, in reference to an instrument, validity is defined as the extent to which that “particular instrument in fact measures what it purports to measure” (Cohen et al., 2007, p. 133).

For ensuring the rigour of the tests' adaptation and translation processes, two actions were taken, as follows:

- i. Two different certified CA tutors checked that the new questions were fair and tried to evaluate the same reasoning pattern as their original counterparts
- ii. I translated the tests from English to Spanish and then asked another native Spanish-speaker to retranslate my translation to English, in order to check if the resultant English version was the same as the original English one.

Regarding the administration process, I tried to ensure that all the evaluation conditions were the same in the two Universities. In order to do so, I took the following steps:

- i. I shared out one test sheet per student
- ii. I read all the questions aloud and made sure that they were clear to all the students
- iii. I encouraged them to ask all the questions they needed in order to finish the test successfully
- iv. Questions related to how to solve the problems were not answered.

Evidently, it is impossible to ensure with absolute certainty that the results are valid and reliable but, given the procedures previously described and the observation of students' reactions to the test, I am sufficiently confident to consider that the results are a relevant starting point for exploring prospective Chilean teachers' reasoning skills through these kinds of tests.

### *Bergling's test*

The items developed by Bergling (1998a) were sent to the Swedish International Association for the Evaluation of Educational Achievement (IEA) committee for validity evaluation. The process aimed to examine whether the items were consistent with the natural sciences, if they were coherent with the reasoning patterns described by Piaget in his theory, and for review of the language and its clarity. Once the inspection was concluded, certain items, figures and choices experienced minor modifications (Bergling, 1998a).

The correspondence between the constructs measured by Bergling's items and the Piagetian experiments was assessed by the following process. A sample of children who were 11 to 17 years old ( $M = 13.9$ ,  $SD = 1.46$ ) took the test, and were also evaluated via the clinical method using three Piagetian experiments (the pendulum, balance scale and chemical bodies experiments). The correlation ( $\rho$ ) between the developmental stage deduced from the two measures was moderate to high and was statistically significant ( $.47 p < .05$  pendulum;  $.60 p < .01$  balance scale;  $.73 p < .01$  chemical bodies) (Bergling, 1998a, p. 177). Although correlation coefficients support the idea that Bergling's test might be considered as a valid method for assessing formal operational thinking, the specific number

of students classified according to each category was not reported in any of the papers published by Bergling (1998a, 1999b, 1999a).

I do not have information regarding the reliability of Bergling's test, since there is no record that suggests that the test was administered to the same or similar samples on two different occasions in order to check if the data obtained were similar.

### *Science Reasoning Task (SRT) test*

A similar procedure to the one used by Bergling was conducted by Shayer et al. (1981) with the purpose of examining SRT validity. Various samples of students, aged from 14 to 15 years, were evaluated by both the SRT tests and six Piagetian tasks, in order to explore if both results would classify students as being at the same reasoning stage. Pearson's correlation was carried out and the coefficients obtained varied from 0.546 to 0.85 (Shayer et al., 1981). It is important to note that the SRTs were validated with students that were aged between 14 and 16. For this reason, it was plausible to hypothesise that my sample would perform better than Shayer's samples (1976, 1978), because my students were older and were therefore more likely to have already developed formal skills.

In order to evaluate this assumption, Table 15 presents the performance of different age samples in the Volume and Heaviness task (SRT). From the data presented in the table, it may be concluded that the Volume and Heaviness task (SRT) is a valid task for evaluating samples older than those originally tested by Shayer (1976, 1978), since older students did not necessarily perform better than did younger ones. Although this conclusion has to be considered carefully because I adapted the task to a non-experimental format, which could have negatively influenced the students' achievements, it is reinforced by those studies that have used SRT for evaluating the formal reasoning skills of older samples of students (Rogan and MacDonald, 1983, Hautamäki, 1986, Prophet and Vlaardingerbroek, 2003, McCormack et al., 2010).

In turn, reliability was assessed by administering the test to a sample of 240 students on two separate occasions. The test-retest correlation found was 0.84 (Shayer et al., 1981). Based on the aforementioned evidence, it is possible to consider the SRT test as a valid and reliable measure of formal operational thinking.

**Table 15: Piagetian stages distribution**

		Age average			
		18.8 yrs	14 yrs	15 yrs	16 yrs
		Tornero (2011)	Shayer (1978)	Shayer (1978)	Shayer (1978)
Thinking stage	Notation	%			
Early concrete	2A	5.5	3.4	3.2	3.8
Mid concrete	2A/2B	32.7	14.2	10.6	11.6
Mature concrete	2B	21.8	28.1	25	32.2
Early formal	2B/3A	30.9	29.8	36	30.3
Mature formal	3A	9.1	23.8	24.3	21.5

### *Tests' difficulty*

As I explained at the beginning of this chapter, one of the pilot study's purposes was to examine if it were possible to use a paper and pencil test for assessing prospective teachers' formal reasoning skills. The evidence presented so far suggests that, at least in terms of the tests' structure and item construction, they are valid and are sufficiently reliable to use as a measure of thinking skills development.

Having explored the tests, it is now important to compare them based on student achievement in each. Therefore, I will present two different sets of data that aim to characterise both instruments in terms of their difficulty. Table 16 shows the 21 items sorted by the percentage of students that answered them correctly. As can be seen, of the 10 items for which less than one third of the



students responded correctly, 6 correspond to Bergling's items and 4 to SRTs' items. In turn, with regard to the six easiest items for which more than 50% of the students answered correctly, three correspond to Bergling's test and three to SRT.

**Table 16: Items' difficulty ranking**

Question n°	Tested by	Students that answered correctly	%
15	Bergling	4	7.3
9b	SRT	5	9.1
8	SRT	7	12.7
2	SRT	9	16.4
14	Bergling	10	18.2
18	Bergling	11	20
20	Bergling	15	27.3
13	Bergling	16	29.1
3	SRT	17	30.9
10	Bergling	17	30.9
17	Bergling	21	38.2
6	SRT	21	38.3
7	SRT	24	43.6
11	Bergling	26	47.3
19	Bergling	29	52.7
16	Bergling	32	58.2
4	SRT	34	61.8
9a	SRT	36	65.5
5	SRT	37	67.3
12	Bergling	49	89.1

Another approach for comparing both tests is to analyse students' overall performance in each one (see Table 17).

**Table 17: Tests' average performance**

Test	Questions	Average % of students that answered correctly
Science Reasoning Task	1-9b	38.40
Bergling's test	10-20	38.03

Based on the information provided, it is possible to state that the tests are very similar in terms of difficulty. This makes it reasonable to assume that the results would have been comparable using either. Therefore, the criteria that I took into account when deciding which to use in the intervention study were related to other factors, such as the interpretation of results or convenience.

In specific terms, there were four main reasons for choosing SRT instead of Bergling's test for my main study, as follows:

- i. It was validated using a larger sample of students
- ii. It has been more widely used in various countries and with older samples worldwide
- iii. The scoring and categorisation process is specified in the instruction manual and is directly transferable to the new test context
- iv. The format seemed more familiar to and was clearer for the students, since most of the questions they asked during the administration were related to Bergling's section of the test.

## *Discussion*

In general terms, the data collected during my pilot study led me to two main conclusions. Firstly, I realised that I needed to consider the role of the researcher very carefully when selecting and adapting the activities. As can be seen in the first part of this section, in which I described the process of choosing and transforming the CAME activities, my attention was originally oriented

towards creating suitable sessions for prospective teachers, rather than on activities that I would present to them. However, during the pilot test, I became aware that the success of the programme depends on both them and me. Therefore, I needed to feel confident and comfortable with all the activities that I was going to conduct and, at the same time, the activities needed to have the appropriate difficulty level in order to foster cognitive challenges and to engage prospective teachers in the tasks. In order to strengthen this in the main study, I sought further training and familiarity with CAME activities through my links at King's College London.

In addition, I concluded that it might be better to develop the programme with older students. At the beginning, I thought that the best thing to do was to carry out the activities with first or second year Bachelor of Education students, since they have not yet developed all their practices or beliefs about teaching. In other words, the sooner I intervened in their initial training as teachers, the better my chances of having an impact on the trajectory of their learning and their approach to it. However, two factors made me reconsider this belief. In the first place, first and second year university students have not had any undergraduate experience of mathematics, so their most recent maths memories are from their school years. As a result, many of them showed a kind of math phobia that predisposed them negatively towards the activities, even though in the near future they would be the ones in charge of teaching mathematics. Thus, my hypothesis was that they had not had the opportunity to reinterpret or re-explore maths in this new and more professional context.

Finally, I noticed that second year students not only had a more mature attitude towards participation and discussion during the sessions, which is a central prerequisite for the adequate development of each CAME activity. Younger students were shyer and less confident in expressing their points of views, especially when they were not completely certain of the particular mathematics problem. By contrast, second year students were more open to participation, to expressing their opinions and to discussion. In addition, second year students had participated in more professional internships so their knowledge and experiences were not only theoretical. In this sense, I feel that it is important to carry out the intervention with a student sample that is motivated and

seemingly more capable of completing and attempting the challenging problems within the activities and to reflect on the importance of the cognitive acceleration approach to their teaching practice.

The methods used in the pilot study and the results found informed the design of the main study. The next section will describe the research design, and the methods and research tools used during the main study.

#### *D. Main study: Research design*

The only information I have concerning the application of a CA programme in a relatively similar context to the Chilean one is the case of Colombia (Uribe and Solarte, 2007, Uribe, 2009), which applied CASE in three secondary schools. However, as I said before, the novelty of this particular project not only resides in the fact that it was applied in a completely different context, but also that it uses prospective teachers instead of school students. Based on the lack of previous evidence from similar contexts and/or students, this research not only carried out the aforementioned pilot study, but also was intended to be exploratory.

Exploratory designs are traditionally used when the research focus is relatively new and the researcher is looking for a novel understanding and construction of meaning (Burns and Grove, 1987, Brink, 1989). According to Brink (1989), exploratory studies usually require the involvement of the researcher in the context of the study and there is little control over the emerging data. For that reason, the researcher has to be flexible and reflective in order to plan future actions based on what s/he has observed in previous stages. This often results in an iterative process whereby the researcher goes back and forth when trying to understand the participants' points of view and meaning making.

In the following section, I present the four research techniques I used, with the purpose of addressing this new research phenomenon and its complexity.

Reflection and critical thinking have become two prerequisites of successful professionals in almost every area of specialisation (Varner and Peck, 2003, Thorpe, 2004). This is particularly true in the case of teachers and teacher training. According to Black (2000), “reflection enables individuals to reframe, reinterpret, and articulate their understandings and beliefs, on a continual basis, in light of new experiences and information. It is a process that encompasses all time designations – past, present, and future - simultaneously. Reflection allows teachers and other professional educators to examine past and present actions and to generate knowledge that informs future actions”.

However, developing a reflective process that is meaningful and which enriches practice is often a painful and complex process for teachers. Pultorak and Barnes (2009) explained this difficulty by referring to four different factors. These are:

- i. Teacher education programmes do not devote enough time to reflection and often do not have structured opportunities to do so
- ii. Teachers lack the ability to look at school-based experiences and to learn about them
- iii. School supervisors are ill-prepared to coach these kind of skills
- iv. Teacher education programmes' workloads are usually not oriented to promote these abilities in prospective teachers.

Many education professionals (Black et al., 2000, Langer, 2002, Varner and Peck, 2003, Thorpe, 2004, Creme, 2005, Sutton et al., 2007) have considered learning journals to be a valuable learning tool that promotes reflective and metacognitive skills. Hedlund et al. (1989) defined learning journals as autobiographical or personal documents in which the learner writes regularly about his or her reflections, ideas, experiences and events. In more pragmatic terms, “A journal is one type of writing assignment that requires the writer to think about something, and to record his/her thoughts about it” (Park, 2003, p.184).

As the most important research objective was to explore prospective teachers' perceptions of the CAME programme in relation to teaching and learning, and because the CAME course in general is oriented towards developing thinking skills, I considered that writing learning journals would be a powerful learning and research tool. On one hand, it could help students to reflect on their own learning processes and, on the other hand, it could allow me to be part of that learning process since they probably would not share all their thoughts with me if they did not write them down.

For these reasons, after every other session, I asked prospective teachers to write down their reflections, prompted by the following questions:

- i. What did you learn?
- ii. What were the most difficult and easiest parts?
- iii. Can you see any of these ideas translated into your own classroom?
- iv. What is the sort of thinking that these kinds of activities promote or require?

I decided not to ask them to write in their learning journals in every session, because all the sessions were very similar in terms of structure and methodology; thus, I was afraid of receiving short and superficial reflections if they felt that they were writing similar things every week.

### *My role as a teacher researcher: CAME sessions' field notes*

Teachers conducting research on their own classrooms is a frequent and well documented research tool (Baumann, 2001, Loughran, 1996, Nolen, 2007, Peeke, 1984). One of the benefits teacher research provides is that they are already 'insiders' in the contexts they are conducting their researchs, so teachers do not have to start by constructing relationships with the participants before the research is being conducted. When referring specifically to the advantages of the dual role of teacher and researcher in the context of teacher training, Loughran (1996) claims that the researcher has the unique and valuable opportunity to

observe the evolution of teachers' change during the process, which was actually the purpose of this particular study.

Despite the benefits and practicalities of this kind of research technique, it also has many risks related to the ability of the teacher researcher to remain critical and reflexive on their own practice and the results being found. Similarly, Kawulich (2005) suggest that teacher researchers should try to avoid becoming an insider and, as a result, to lose impartiality and the ability to write analytically and critically about the cultural aspects of the context. Therefore, the position the teacher occupies within the classroom context should maintain a balance between being an insider in the sense of being part of what is going on in that particular setting while, at the same time, being an outsider who is able to maintain a certain distance in order to observe, describe, analyse and understand the particularities of that context.

In the case of this research, one evident challenge was the position of power that was inevitably connected to my role of teacher researcher. As this asymmetrical relationship cannot be denied or eliminated, a good way of dealing with it was to follow certain ethical procedures (Marshall and Batten, 2004) that allowed me to ensure that their participation in the research project was voluntary and informed. This point will be further developed in Chapter V, p. 142, which deals with ethics.

The role I played during CAME sessions and the field notes I wrote evolved during the data collection stage. As Kawulich (2005, p.8) states, "It is important in the early stages of the research process for the researcher to make accurate observation field notes without imposing preconceived categories from the researcher's theoretical perspective, but allow them to emerge from the community under study". Therefore, during the first CAME sessions, I wrote field notes that were more general and descriptive; while later I became more analytical and hypothetical.

However, from the beginning, I used the type of observation field notes that Angrosino and DePerez (2000) call 'selective', which consists of deciding the topics, activities or aspects of the situation on which the researcher will focus. Based on my research questions, I wrote down various questions that could help

me to focus my attention during the lessons. These were:

- i. What did they find easy?
- ii. What did they find difficult?
- iii. How did they engage in the activities?
- iv. How did they work in groups?
- v. Was someone leading the discussion, or did they all contribute to the discussion?
- vi. Did they manage to build their comments on someone else's comments?
- vii. Did they pay attention to each other?

Finally, in order to ensure the rigour of my field notes, I always wrote them within 24 hours of the session and I always came back to them before the next session to make comments and to highlight certain issues that I wanted to stress or to be particularly focused on during the next session. As a result, the process of writing my field notes was essentially an iterative and analytical process.

## *Interviews*

Interviewing is one of the most frequently used methods of data collection in education and other subject areas (Dilley, 2004). According to Baker and Johnson (1998), “the most conventional perspectives on interviewing in social science and educational research are those which treat the interview as a method of data collection, and the contents of answers as the data. Interviewees are asked to provide reports or descriptions about interior states or external events in the world outside the interview. This methodology often presupposes that some reality — knowledge, beliefs, stories, perspectives — pre-exists the interview. This approach provides for coding-based analysis of ideas and themes in the interview transcripts”.

From the above description, it can be seen that the term ‘interview’ is quite broad and it could be used in a variety of contexts and with multiple purposes



(Cohen et al., 2007). In the case of this specific research, the rationale behind interviewing was to explore my participants' perceptions and processes of change in-depth and from their own points of view. The interviews were semi-structured in order to fulfil two purposes at the same time, namely to create a flexible interview context in which the interviewees had a certain freedom to discuss aspects and topics that seemed relevant and interesting to them, and to guide the interview towards precise topics that were relevant for answering the specific questions of this research.

I had the initial idea of interviewing three people from each group: 1 high achiever, 1 average achiever and 1 low achiever in terms of their scores on the Science Reasoning Task test, at the beginning and at the end of the course. I had thought that the selection of the interviewees would be based on prospective teachers' performances in the pre-test, in order to have variability in my sample and to be able to explore the experience of students with different levels of achievement during the CAME course. However, once I started the course, I realised that, as the course was voluntary, the participation was potentially volatile. Given this, I could not take the risk of interviewing three people at the beginning of the course, as they may not be participating in the course by the end of it. For this reason, I decided to interview four people from each group in order to maximise my chances of them still being there at the end of the course and of being able to investigate people who had completed the entire course. In addition, I took into consideration the level of attendance when deciding who to interview, since I thought this was the best way of evaluating their commitment to the course and of increasing the chance of having the same people at the end of the term.

In the end, I interviewed four prospective teachers from UA University, four from UB University and only three from UC University, because the entire experimental group had four participants. Of these 11 interviewees, only one gave up the CAME course before the end thereof (for medical reasons), so I did not have an opportunity to conduct a follow-up interview with her. In total, I conducted 21 interviews: 11 pre-intervention interviews and 10 follow-up interviews. For more details regarding the pre- and post-interview protocol, please refer to Appendix E.

### *Formal reasoning skills test: The Science Reasoning Task (SRT)*

The Science Reasoning Task (SRT) test was used as a pre- and post-intervention test, with the intention of exploring if it were possible to observe changes in terms of measured reasoning skills in the research participants. As I argued before, I was not particularly optimistic about finding positive results for this test, because the intervention was shorter than are regular CA programmes. However, it was interesting to explore this aspect and, just in case I were to find results, I decided to have a control group for each of my intervention groups in order to compare the difference between the pre- and post-tests. Otherwise, I would not be able to attribute the potential effects to the CAME programme.

Anyone familiar with the SRT would raise the question of why I decided to use a science test if my intervention were to involve mathematics. This is a completely legitimate point and I will present the three main reasons for my decision. Firstly, cognitive acceleration programmes have usually been considered to affect the development of general thinking skills (Adey and Shayer, 1990, Adey and Shayer, 1994, Adey and Shayer, 2002). Therefore, even though I did not plan to evaluate transferability or the impact of the CAME course on students' achievements in other subjects, if there were an effect in terms of formal thinking skills, that effect should be observable in a test that attempts to measure formal thinking skills via science-related tasks.

The second reason was that using a maths test could have interfered with the results. Even though the main intention of CAME is not to teach specific math content but to develop thinking skills, students make use of their previous knowledge to solve each problem or situation. Consequently, I did not want the students to use algorithms that they could have seen during lessons when answering the test, because that would not be a signal that the course enhanced their general reasoning ability, but would only reveal their mathematical knowledge and/or skills.

Finally, the SRT tests (Shayer, 1977), together with the Thessaloniki test (Demetriou et al., 1991), are instruments that have always been used for assessing the impact of each CA intervention (Adey and Shayer, 1990, Adey and Shayer, 1994, Shayer, 1996, Adhami et al., 1997, Shayer et al., 1999, Adey et al., 2002, Adey and Shayer, 2002, Shayer and Adhami, 2006, Shayer and Adhami, 2007). Of the two, I chose the SRT because it has been more rigorously validated and, more importantly, it has been widely used in different contexts and countries (Rogan and MacDonald, 1983, Prophet and Vlaardingerbroek, 2003, Mohapatra and Mahapatra, 1998, McCormack et al., 2010), which is highly relevant considering that I was planning to use the test with an older population and in a language other than English.

The following table, Table 18, shows how the different research techniques were used and distributed during the data collection stage that lasted for 17 weeks. Dark squares show the research tools that were present in each week. In addition, Table 19 shows a summary of the data collection stage by establishing the relationship between the research questions that this research answered and the research methods that were used to do so.

**Table 18: Data collection time-frame and research techniques**

Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Formal Reasoning Test																	
Learning Journals																	
Field Notes																	
Interviews																	

**Table 19: Data collection planning**

<b>Main Research Question</b>	<b>Specific Research Questions</b>	<b>Research Objectives</b>	<b>Data Collection techniques</b>
How do prospective teachers perceive a cognitive acceleration approach in relation to teaching and learning?	How do the formal reasoning skills of prospective teachers change after experiencing a course of cognitive acceleration activities?	To explore how prospective teachers view a cognitive acceleration approach in terms of experiencing the activities as learners.	Science Reasoning Tasks Learning Journals Field Notes Interviews
	How do the views of prospective teachers towards teaching and learning change following a cognitive acceleration intervention programme?	To explore how prospective teachers view a cognitive acceleration approach in terms of the application of the activities to teaching and learning mathematics	Learning Journals Field Notes Interviews
		To explore how prospective teachers' views regarding the importance of thinking change following a cognitive acceleration intervention.	
	How do the attitudes of prospective teachers develop following an intervention that uses a cognitive acceleration approach?	To explore how prospective teachers view a cognitive acceleration approach in terms of their confidence in teaching mathematics.	Learning Journals Field Notes Interviews

### *E. The sample and the sampling process*

The population for my research was prospective primary teachers in Chile who were in their fourth or fifth (last) year of their Bachelor of Education programmes. The reason behind restricting the potential population to fourth- or fifth-year students was that what I wanted to explore was not only the impact of the CA approach on their reasoning skills and perceptions, but also how they perceived the approach in relation to teaching and learning mathematics or, in other words, how useful they thought the approach to be for teaching and learning mathematics. In this sense, it was better for them to have had some internship practice in order to reflect about the approach in relation to their experience as future teachers, and not only as students.

As the course was going to be delivered during an entire university term which, in Chile, lasts for five months, I decided to design it as a formal course that could meet the requirements of the Education Departments in order to propose it to some of them and to offer it as one of the optional courses that prospective teachers could take during that term. Had I not done so, it would have been highly unlikely that I would be able to recruit the participants that I needed and to ask them to come once a week for an entire term. In addition, the initial idea of using a CA approach for training prospective teachers in Chile started with my own conviction that it could be a significant contribution to the way in which teachers are being trained in Chile (for more details, see Chapter II p. 21). For that reason, it was also strategic to involve Education Departments in the development of the project in order to be able to rely on their commitment and to maximise the potential impact of the course.

Based on these reasons, the specific sampling method I used was an intentional cluster sampling (Hesse-Biber, 2010), whereby the smallest eligible unit was Education Departments and not individual students. This type of method had the advantage of allowing me to choose Universities that could be a good representation of the variety of higher education institutions that exist in Chile, and to ensure that participating Universities were committed to the project, because all the sessions were going to be developed in their lecture rooms during

term time and they had to provide all the necessary materials (blackboards, pens, photocopies and so on). Therefore, it was essential that Education Departments were engaged in the project in order for them to facilitate such resources.

Even though the sampling process had the aforementioned advantages, it also had the drawback of generating samples composed of smaller groups (clusters) that, in turn, were composed of individuals who shared certain characteristics. For example, all the participants who are studying at the same University are likely to have similar academic backgrounds, because each University has specific entry requirements. This makes it difficult to ensure that the sample selected is representative of the population; that is to say, that the individuals included in the sample are a good illustration of the variety present in the general population (Cohen et al., 2007).

**Table 20: Participating Universities characterisation**

University	Selectivity level	% of students that come from each type of school		
		%	Private	Subsidised
UA	57	92	6	2
UB	24	67	25	7
UC	1.25	2	43	55

(Brunner, 2009)

Three Departments of Education participated in this research. For confidentiality reasons, they will be indicated as follows: (i) UA, (ii) UB and (iii), UC. I decided to include only three universities for reasons of feasibility. As I had to deliver all the CAME lessons and analyse all the data collected from them, three was a large enough number to ensure variability within my sample, while still being small enough to be manageable within the resources and time frame of a PhD thesis. Although all the participating Universities are private and do not receive public funding, the profile of the students they accept is quite diverse, both in terms of previous academic achievement and in terms of socioeconomic background (see Table 20). It is important to clarify that, in Chile, the type of

school (private, subsidised or public) that a student attends provides information regarding his/her socioeconomic background. In this sense, wealthy families usually choose private schools for their children (8%), while middle class students are typically enrolled in subsidised schools (54%) and the most disadvantaged students attend public institutions (38%) (Ministry of Education, Government of Chile, 2012).

Two of the three participating universities (UA and UB) had participated in the pilot study one year before. However, the participating students were not the same, since the pilot students had graduated at the end of the previous year. For recruiting individual students, I sent an email to the secretaries of the Departments of Education and they forwarded the invitation to the potential participants. As a response to that invitation, a group of students from each university registered for the course (see Table 21). However, only a group of them actually participated in it (see Table 22), which means that they attended to at least 60% of the CAME sessions.

**Table 21: Students registered for the course**

	UA	UB	UC	TOTAL
<b>N° students in the 1<sup>st</sup> session</b>	24	11	10	<b>45</b>

**Table 22: Participating students per University**

	UA	UB	UC	TOTAL
<b>Experimental</b>	14	8	4	<b>26</b>
<b>Comparison</b>	4	8	3	<b>15</b>
<b>TOTAL</b>	<b>18</b>	<b>16</b>	<b>7</b>	<b>41</b>

In each university I worked with, there was one group of students as an experimental group and one group of students as a comparison (see Table 23).

**Table 23: Experimental and comparison group characterisation**

	Experimental	Comparison
<b>Male</b>	1	4
<b>Female</b>	25	11
<b>Average age</b>	23.2	22.9

After obtaining permission to work in the three Universities, I had to recruit individual students to participate in my course. In this sense, even though the Universities had agreed to be part of the project, the students were completely free to participate or not. The fact that the students voluntarily attended the course might imply that my sample was biased in the sense that it did not represent the entire population of students at that University. My sample was probably composed of students who were more motivated or of students that had a particular interest in participating in the course. When conducting research, the researcher always has to make decisions and compromises and, in this case, it was not possible for ethical reasons to select the participants randomly. As a result, I had to accept that my sample could be slightly biased and the implication that this could have for my results.

#### *F. The selection, adaptation and implementation of the CAME activities*

The number of activities selected was guided by the number of weeks (17) that constitute a semester in Chile. As a result, the planning of the course included 12 CAME lessons and two evaluative lessons, one at the beginning and one at the end of the course. In addition, in order to decide which of the 30 CAME activities would be included in the course, I took three criteria into consideration:

- i. The appropriateness of the activities for the group of students with which I was going to work, not only in terms of age but also in terms of characteristics. In order to fulfil this criterion, the role of the pilot



study that tried out the activities one year before the main study was crucial.

- ii. The coverage of the six different strands that are included in CAME lessons (Number system and properties, Multiplicative relations, Functions, Algebra models, Shape and Space and Data handling).
- iii. The inclusion of activities with different difficulty levels based on the Piagetian levels described by each activity. It was important to cover the entire range of Piagetian levels because, in the pilot, I had already identified that prospective teachers had different levels of formal reasoning skills; thus, some of them were operating at the concrete level and others at the formal level.

Based on these three points, the sixteen CAME activities selected were the ones stated in Table 24. This table shows the name of the activities selected, the main strand of focus (black circle), the secondary strands of focus (white circle) and the range of Piagetian levels covered by each. The recognition that some strands were more frequent (Data handling) than others (Number system and properties) is explained by the fact that, in the full set of CAME activities, there are some strands that are more frequent than are others.

Once I selected the activities, the next phase consisted of adapting them to the appropriate context. It is important to remark that the adaptation process did not finish when, before the beginning of the term, I had designed and planned the entire course. As was the case during the entire research process, the adaptation was also iterative. In this sense, based on the experience and results of previous activities, I adapted the next ones because, as the course proceeded, I started to realise which aspects were more successful and which generated more fruitful reflection amongst my students.

**Table 24: The CAME activities selected, their strands and Piagetian levels**

Lesson	N u m b e r  s y s t e m  a n d  p r o p e r t i e s	M u l t i p l i c a t i v e  R e l a t i o n s	F u n c t i o n s	A l g e b r a  m o d e l s	S h a p e  a n d  S p a c e	D a t a  H a n d l i n g	Range of Piagetian Levels 2B 2B* 3A 3B 4 5 6 7/8
Furniture design	○				○	●	5 - 6
Sam and the newspaper						●	5 - 6
Which offer shall I take?		○	●	○			4.5 - 6
Chocolate box		○	●	○			5 - 7
Circle functions		○			●		4.5 - 6.5
Three dice		○				●	5.5 - 6.5
Prediction and correlation						●	5.5 - 6.5
Accuracy and errors	●	○					5 - 6
Heads and tails I and II						●	5 - 6
Expressions and equations	○			●			5 - 6
Comparing correlations		○				●	5.5 - 6.5
Rates of changes		○	●	○			5 - 6
Data relations	○					●	5.5 - 6.5
Chunking in algebra			○	●			5 - 6
Accelerating the acceleration			○	●			5 - 7
How do I handle the data? I and II		○	○			●	5.5 - 6.5

(Adhami et al., 1998)

### *G. Ethics*

In this study, I followed the King's College London guidelines for good practice in academic research. This project, REP (EM)/10/11-44, received full ethical approval from the Education and Management Research Ethics Panel on the 12<sup>th</sup> of May 2011.

As the CAME course was offered as one of the optional courses that prospective teachers could take during the term, it was very important for my students to understand the difference between participating in the course and participating in the research project. Even when some of them were not interested in taking part in the research, they were still allowed to take the course. In order to explain this, I gave them an information sheet during the first session (see Appendix F) that stated that this course was part of a research project and that voluntary participation in it involved the following:

- i. Answering a multiple-choice and open-ended question test at the beginning and at the end of the course
- ii. Writing learning journals every other session of the course
- iii. Possibly being invited to participate in an interview that would last no more than 60 minutes at the beginning and at the end of the course

I explained to the prospective teachers that their participation in the research was purely voluntary. In the information sheet, I also explained that they were free to choose not to answer the test questions or to complete the learning journals. In addition, they had the right to leave the course or the research project at any time and to withdraw all their information from the study before November 2012, when the data analysis stage was scheduled to begin. Apart from accepting the terms stated in the information sheet, they had to sign a consent form as a way of formalising their agreement to participate in the project (see Appendix G). Both documents clearly expressed that they had the right to withdraw their participation before November 2012 without experiencing any consequences.

Some students who were contacted and invited to participate did not agree to take part in the study. I did not report or discuss any individual's attendance

with the university tutors. Student teachers who participated only in the formal reasoning test (control group) also had the right to withdraw their participation at any point before November 2012, a date that was clearly stated in the information sheet and consent form, when I began to work on the analysis of the data.

The confidentiality of the participants in the formal reasoning test was assured by not disclosing the results to any university authority or tutor, and the data were kept in files secured by a key to which only I had access. Since the test I was using was not infallible and the students knew that I was testing their reasoning abilities, I decided not to disclose their test results to them either, as so doing could damage their self-esteem and I was concerned about the detrimental effect this could have. Therefore, I stated in the information sheet that they would not have access to either their test results or the test's answers.

I also let the students know that one of benefits of participating in the research was the possibility of developing or strengthening higher order thinking skills. Another benefit related to the abovementioned was the opportunity to affect their academic achievement in a positive manner. Previous evidence using school students showed that cognitive acceleration programmes have long-term and far transfer effects on academic performance (Shayer, 1996, Shayer and Adhami, 2006). Finally, through their participation in the programme, the students would have the opportunity to experience and to increase their knowledge of thinking skills, which could be useful for their future teaching careers. The information sheet also stated that all the data was to be used once only and for the purposes described in the consent form. Therefore, if I want to use that data again in the future, I would have to ask them to re-consent.

With regard to the interviews, I contacted the potential participants and gave them an information sheet (see Appendix H) that stated the purpose of the research, the nature of their participation and a consent form (see Appendix I) in which they could agree to being interviewed. Both documents clearly expressed that they had the right to withdraw their participation at any time without experiencing any consequences. The information sheet also gave them basic information regarding the length of the interview, the purpose thereof and that it

would be audio recorded and transcribed verbatim for data analysis purposes. It also stated that it would be destroyed after the completion of the analysis.

## *H. Data Analysis*

### *Content analysis*

All the written material, which included interview transcripts, field notes and learning journals, was analysed using a qualitative content analysis. Hsieh and Shannon (2005) define a qualitative content analysis as “a research method for the subjective interpretation of the content of text data through the systematic classification process of coding and identifying themes or patterns”. They also claimed that content analysis is a useful research method when the researcher wants to explore or describe a phenomenon that has a relatively limited theoretical and literature background, as is the case in this research.

Although some authors (Berelson, 1971, Ryan and Bernard, 2000, Babbie, 2001) have considered content analysis to be an eminently quantitative research method, it has become increasingly popular as a qualitative one. One of the main features that is usually mentioned as a distinction between quantitative and qualitative content analysis is that the latter not only considers the explicit meaning of the text, but also the latent meaning (Mayring, 2000). With regard to highlighting the interpretative dimension of the qualitative content analysis, Bryman (2004) stated that it is “An approach to documents that emphasizes the role of the investigator in the construction of the meaning of and in texts. There is an emphasis on allowing categories to emerge out of data and on recognizing the significance for understanding the meaning of the context in which an item being analyzed (and the categories derived from it) appeared”.

According to Kohlbacher (2006), content analysis is a cycle that should involve three different types of analysis in order to be performed thoroughly and in depth. These are:

- i. *Summary*: the categories created during the analysis are oriented not only towards diminishing the amount of data, but also towards creating units that maintain the original meaning and which can be related to each other. With regard to this point, Weber (1990) stated that the ultimate purpose of content analysis is to explore written language thoroughly, with the intention of reducing large quantities into smaller categories that conserve the speaker's meaning and intention
- ii. *Explication*: given that the ultimate intention is profound understanding, it is necessary to relate emerging categories to their contexts in order to explain and clarify the data
- iii. *Structuring*: this refers to the creation of a text structure. In order to accomplish this, the first decision is to define the unit of analysis, while the dimensions of the structure can only be developed thereafter. According to Rourke et al. (2001), the most frequent units of analysis used in content analyses are, from the smallest to the largest, illocution, sentences (or syntactical units), 'units of meaning' (or thematic units), paragraphs (sections) and the message. The next stage is to create each category and to give examples that illustrate each of them, or at least the most relevant ones. One of the keys to the process of structuring the data is the iterative aspect thereof, since this allows the data to be reviewed repeatedly until the categories are well defined and clearly distinguished. In this sense, not only can the dimensions of the category system be changed, but new categories can also be created and others eliminated.

There are two different ways of creating the category system, either inductively or deductively (Hsieh and Shannon, 2005). When the researchers are following an *inductive* category creation process, they tend not to use previous categories, but to let them emerge from the data. In this sense, instead of defining the categories that will be looked for in the data, they established the criteria for defining categories that delimit the data to be considered in the coding process based on the literature review and the research objectives. By always following an iterative process, the coding stage goes forward and backward until all the

categories have been revised and the set is coherent with the data (Mayring, 2000). On the other hand, when the coding process is applied *deductively*, categories are derived from the literature review and are defined before the coding process starts. Consequently, the coding process consists of methodically applying the created categories to the corresponding text extracts (Mayring, 2000). Figure 11 presents a diagram that illustrates the inductive and deductive processes of coding.

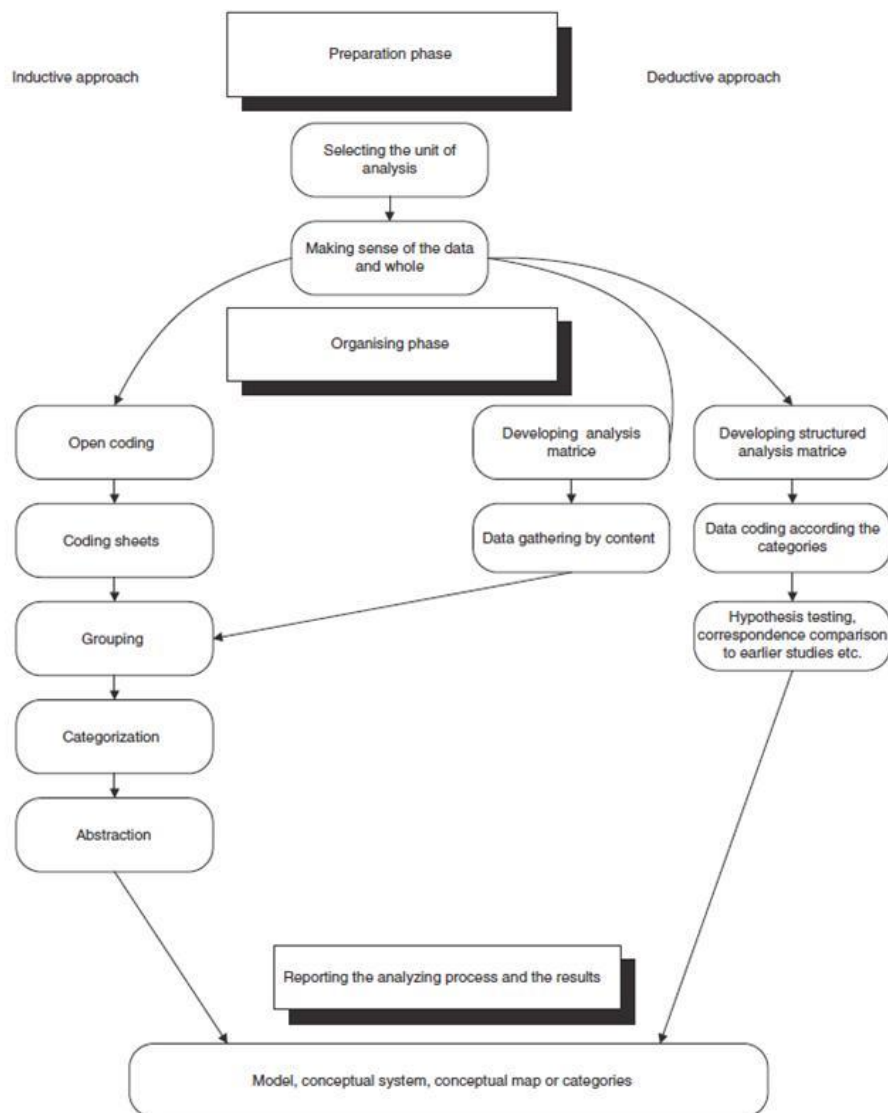
The coding process used in this work cannot be defined as being exclusively deductive or inductive, since a combination of both approaches was used. However, it is possible to say that a deductive approach was predominant. Initially, a list of codes was derived from the literature review and the research questions as a way of guiding the initial immersion in the text. However, that code list was considered to be preliminary, given that it was theoretically driven and the questions addressed by this research were novel; therefore, I could not completely rely on previous evidence. For this reason, the codes from the initial list were treated in the same way as the codes that emerged during the coding process. In other words, they were continuously revised and adapted in the light of new evidence.

The data analysis followed the conventional content analysis process suggested by Hsieh and Shannon (2005). I first read all the transcripts in order to have a complete picture in mind. I then read every word in detail, with the purpose of creating the first emerging codes and linking some text passages to the existing (theoretical) ones, by using qualitative data analysis software (ATLAS.ti, version 5.2). The next step was to re-read everything but, this time, highlighting my impressions and reflections, as well as taking note of thoughts and aspects that were relevant to the initial analysis. Subsequently, based on these initial thoughts, the first labels for the codes emerged and they began to be structured into a preliminary coding list.

As a result, I started to find relationships between the codes, which meant that they were organised into broader categories, or group codes. Each code, subcategory and category was then defined and exemplified. Finally, I tried to develop a structure that reflected the hierarchy of these categories. It is important

to mention that, as stated previously, this was a continuous cycle in which I continuously revised and refined the codes until they formed a coherent and trustworthy representation of the raw data. It is important to clarify that, as the study was conducted in Chile where the language is Spanish, all the raw data was in Spanish and it was never translated into English. However, all the analytical processes were carried out in English. In other words, all the emerging codes, the explanations hereof and the quotations used to exemplify them were developed in English from the beginning.

**Figure 11: Preparation, organisation and the resulting phases in the content analysis process**



(Elo and Kyngäs, 2008).



Table 25 presents a summary of the final code list. For more details regarding the code list and how it evolved during the content analysis iterative process, please see Appendix J.

**Table 25: Final Code List Summary**

CATEGORY	SUB-CATEGORY	CODE NUMBER	CODE NAME	CODE DESCRIPTION
CAME	EXPERIENCE	1	CE_CONFIDENCE	More confident about teaching maths
		2	CE_MATHSKILLS	They realised they have maths skills
		3	CE_METACOGNITION	They are more aware of their own learning/thinking processes
		4	CE_MOTIVATION	The course was motivating
		5	CE_THINKINGSKILLS	They had to use their own reasoning skills to solve the problems
	IMPROVEMENTS	6	CI_APPLY	They did not put what they had learned into practice
		7	CI_STUDENTS	It would have been better to have more classmates participating
		8	CI_TIME	The course or the sessions were too short
	METHODOLOGY	9	CM_CONSTRUCTIVIST	The course methodology was constructivist
		10	CM_DIVERSITY	The course emphasised a flexible approach to the problem
		11	CM_NOVELTY	The methodology we used during the course was new to them
		12	CM_SHARING	Having to share was useful in terms of learning from their peers
MATH	LEARNING	13	ML_ABILITY	Being able to learn maths is related to a general ability
		14	ML_CONCRETE	Learning maths should be concrete
	TEACHING	15	MT_BASIC	Teaching primary maths is basic, simple or 'easy'
		16	MT_COMPLEX	Teaching maths is complex

		17	MT_CONF	They feel confident about teaching maths
		18	MT_INTEGRATED	Maths should be taught as an integrated subject
		19	MT_MECHANIC	The process/mistakes should be emphasised more
		20	MT_NOCONF	They do not feel confident about teaching maths
		21	MT_USEFUL	Teachers should help students to realise that maths is useful
	EXPERIENCE	22	ME_BAD	They have had previous, bad experiences with mathematics
LEARNING	VIEW	23	ME_GOOD	They have had previous, good experiences with Mathematics
		24	LV_ACTIVE	Students should play an active role in their learning processes
		25	LV_CONCRETE	They refer to learning as being concrete
		26	LV_EMOTIONS	A key to learning is promoting positive emotions in students
TEACHING	FEELINGS	27	TF_CONF	They feel confident about teaching in general
		28	TF_NOCONF	They do not feel confident about teaching in general
	VIEW	29	TV_CHALLENGE	Teachers should challenge students and should make them think
		30	TV_CONTRADICTION	There are contradictions regarding their teaching views
		31	TV_EXPECT	Teachers should have high expectations of their students
		32	TV_MECHANIC	Teachers usually teach in a mechanical way
		33	TV_MEDIATOR	Teachers should be mediators of their students' learning
		34	TV_STUDNEED	Teachers should identify their students' needs
THINKING	VIEW	35	THV_APPLIED	Thinking is being able to apply prior knowledge to new areas
		36	THV_COMPLICATED	They say that is difficult to define thinking
		37	THV_KNOWLEDGE	Thinking is related to the amount of knowledge a person has
		38	THV_MIND	They view thinking as being the same as mind/intelligence
	TEACHING	39	THT_CONF	They feel confident about promoting thinking skills

		40	THT_LACK	Teachers/schools do not encourage students to think
		41	THT_NOCONF	They do not feel confident about promoting thinking skills
		42	THT_RELEVANT	Teaching thinking is crucial for meaningful learning

## *Validity*

Validity is a term originally used for assessing the quality of quantitative research results. For this reason, many qualitative researchers (p.e. Kirk and Miller, 1986, Lincoln, 1995) have rejected the idea of judging qualitative research based on validity standards. However, I prefer to refer to those scholars that have redefined the term, following the assumptions and claims made by qualitative research.

Based on the recognition that we can only access reality as observers and interpreters and the fact that researchers are part of the reality they are trying to explore, validity is understood as the extent to which the account is grounded in the perspectives of the community under study. In relation to this, Maxwell (1992) claimed that validity is always relative, because it cannot be separated from the perspectives of those involved in the research process. This does not mean that every account is equally valid, but it implies that some accounts are more valid given different perspectives. In this sense, according to Maxwell (1992 p. 284), “Validity is not an inherent property of a particular method, but pertains to the data, accounts, or conclusions reached by using that method in a particular context for a particular purpose”. Brinberg and McGrath (1985) agreed with Maxwell’s definition and pointed out that validity can be considered to be the integrity and quality of the research results in regard to its purposes and circumstances.

In the process of securing validity, several scholars (Patton, 2001, Cohen et al., 2007) claimed that triangulation plays a central role. Patton (2001) defined triangulation as the use of “several kinds of methods or data, including both quantitative and qualitative approaches”. Using different methods or data to

approach the same phenomenon serves the purpose of strengthening the research results and, consequently, their validity. Given the complexity of social reality, it is commonly agreed that it could be better understood by using multiple approaches and various types of data, in order to have a complete picture of the phenomenon under study.

Cohen and Manion (2007) stated that triangulation has at least two significant advantages for the research process, namely:

- i. When different methods are used or different types of data are collected, the researcher has an opportunity to explore whether all of them lead to the same conclusions. As a result, triangulation gives the researcher the necessary confidence to claim that the research conclusions are not the exclusive result of the data collection and analysis method used.
- ii. Increasing the use of different methods will help to defeat what Cohen and Manion (2007) called ‘method-boundedness’, which means that many researchers use only certain methods. Using different methods or/and data might lead to a more comprehensive approach to understanding social and human behaviour.

Based on this argument and on the claims that support the benefits of a multi-method approach reviewed in a previous section (see Chapter V p. 104), this researcher used these guidelines when conducting this research. This is one of the reasons that I decided to use not only different types of qualitative methods (interviews, learning journals and field notes), but also quantitative ones (Science Reasoning Task tests).

One example of the way in which I used different kinds of information to triangulate my data is the following: Before starting to code an interview, I read the complete transcript and wrote a summary that covered the main ideas revealed during the interview with the purpose of not losing the sense of the whole through the coding process. The second step was to code the interview and the learning journal of the same person several times, until I had the impression that the emergent code structure was coherent with the original data. Having done that, I then compared the interview summary, the code structure and my field notes on

that person in order to explore if the conclusions reached by each method independently were coherent. In the event that they were not coherent, I returned to the original data in order to make the necessary adjustments and to ensure that the conclusions were a fair representation of the original data.

### *Reliability*

Some scholars (p.e. Stenbacka, 2001) have argued that the concept of reliability is not relevant for judging qualitative research, as it is in quantitative research. I tend to agree with those authors (p.e. Phillips, 1987, Hsieh and Shannon, 2005) who have claimed that it is an appropriate way of evaluating qualitative research quality, but with its own particularities and standards. In this regard, Golafshani (2003) stated that every researcher, when conducting research, should be looking for reliability at every step, including research design, results analysis and research quality. Within the context of qualitative research, the term reliability is usually understood as being related to credibility, transferability and trustworthiness. In other words, it generally means that the conclusions drawn from the data are truly supported or grounded in them.

Four frequent types of errors that affect reliability when conducting content analysis are described by Krippendorff (2004), as follows:

- i. Those related to the unit of analysis, particularly when it differs from the data
- ii. When the properties are not extensively and clearly defined, there will be disagreement during the coding process
- iii. When the dividing line between two or more categories is too fine
- iv. If the problems with the coding are not related to either i, ii or iii, it is reasonable to think that there is a problem with the coders that might be solved by further training.

When referring to securing validity in a research process that uses content analysis, Kohlbacher (2006) claimed that one of the most important challenges is related to the trustworthiness of the coding. The terms inter-coder and intra-coder

reliability are central to this issue. Inter-coder reliability is defined as the level of agreement between two different coders when coding the same text extract, and is usually expressed as a proportion between 0 and 1, with 0 being null agreement and 1 being perfect agreement. In turn, intra-coder reliability assesses the stability of the coding of one coder (Kohlbacher, 2006).

In order to strengthen the reliability of my results, 15% of the data was doubled-coded. This involved two complementary, yet different, processes. On one hand, two different people coded the same text independently and we then compared, discussed and agreed on a final version of our coding. The second coder was another PhD student from a Chilean university who also needed help with his coding process. Therefore, he helped me with the coding of my data and I helped him with his. This person was sufficiently proficient in English, which was a prerequisite because, as I said before, even though all my raw data was in Spanish, all the data analysis was carried out in English. This process was intended to reduce the bias that might be involved in the coding process of only one coder. I found an 87% correspondence between the two coders, which might lead me to suggest that the coding list had an acceptable level of definition and clarity when guiding the coding process. On the other hand, 30% of the data that was not coded by two different people was coded twice in order to explore the concurrence level of my own coding of the same text on two different occasions. As a result, I found a 92% concurrence between my first and second codings, which is an acceptable level of agreement given the characteristics of the process.

## VI. Results

The main purpose of this study was to gain profound understanding of the processes of change prospective teachers experienced during a course that used the CAME approach. For that reason, in this chapter the sections will be presented in a sequence that will allow the reader to acquire deeper levels of comprehension through each one of them.

Specifically, the chapter will start by showing the results prospective teachers obtained in the Science Reasoning Task test at the beginning and at the end of the CAME course, in order to set the ground for prospective teachers own perceptions that are coming afterwards. Consequently, after all the statistical analysis are presented, the evidence from the interviews, learning journals, and field notes will be provided in order to better understand what are the deeper processes of change that give support and are behind the significant improvements that were observed in the SRT test. Finally, a supporting case will be presented to again construct an even deeper understanding of the processes of change observed in the participants by exemplifying how this change looked like in one prospective teacher: Sara.

### **Evidence from the Science Reasoning Tasks: formal reasoning skills**

#### *A. Prospective teachers' change after the CAME course: Descriptive results*

This section deals with the question of how the formal reasoning skills of prospective teachers changed after experiencing a course on cognitive acceleration activities. The focus will now be on the results that prospective teachers achieved in the Science Reasoning Task tests that were taken in the first and last sessions of the CAME course.

As was described in the methodology chapter (see Chapter V, p. 117), the task II, called *Volume and Heaviness*, was chosen from the Science Reasoning

Tasks as the most appropriate to fulfil the purposes of this research. The Volume and Heaviness task consists of a group of questions and each was classified according to five categories, from early concrete (2A) to mature formal (3A), based on the reasoning level they demand. For more information about the reasoning level that each of the Volume and Heaviness questions demands, please refer to Appendix K.

In turn, each person's performance is classified according to one of those five categories (from 2A to 3A), depending on the combination of questions and their difficulty level of the questions that they answered correctly. As explained earlier, these tasks have been trialled with large cohorts and on many occasions since Shayer and his team originally designed them in the 1970s, and are therefore considered to be reliable (see Chapter V, p. 133). For more details regarding the scoring criteria, see Appendix L.

Following the corresponding scoring rules, the 41 participants (26 in the experimental and 15 in the comparison condition) were classified according to the five reasoning level categories for the pre- and post-tests. In addition, the Volume and Heaviness task provided information regarding the number of correct answers each person scored in each of the tests (see Appendices M and N). Even though the regular format in which the SRT results are usually presented is in the form of reasoning level categories, I felt that including additional information regarding the number of answers that each person got correct could be helpful, not only for checking the reasoning level results produced by the Volume and Heaviness task, but also for providing additional support to the conclusions drawn from it in terms of reasoning level change.

This is why next section will describe not only the reasoning level of prospective teachers at the beginning and at the end of the CAME intervention, but also the number of questions they answered correctly, with the purpose of exploring if there is a relationship between the two measures.



### *B. Relationship between prospective teachers' reasoning levels and the number of correct responses*

Before including the information regarding the number of questions that each participant answered correctly, I ran two Spearman correlations, one for the pre-tests (reasoning level and number of correct responses at the beginning of the CAME course) and one for the post-tests (reasoning level and number of right responses at the end of the CAME course). The Spearman correlation is usually employed when non-parametric data is being analysed, which is the case in this particular analysis (Spearman, 1904, Caruso and Cliff, 1997).

The intention aimed to explore if it were possible to find a positive correlation between the number of correct responses and the corresponding level of reasoning classification. Evidently, if the SRT test is a good measure of an individual's reasoning level, the expectation would be to find a positive and significant correlation between these two measures, since they essentially rely on exactly the same data, but are coded in two different ways. In other words, not finding a positive correlation would bring the classification scheme into question.

The Spearman analysis showed a positive, statistically significant and strong correlation between the two pre- ( $r = 0.794$ ;  $p < 0.05$ ) and post-tests ( $r = 0.666$ ;  $p < 0.05$ ) (for more details, see tables 5 and 6 in Appendix O). These results mean that the participants who obtained higher classifications in terms of their reasoning levels in the pre-test also tended to get higher numbers of correct responses in the same test. Consequently, those who had lower reasoning levels in the pre-test also had fewer numbers of correct responses. The same conclusions are applicable to the post-test.

After establishing that there was correlation between the two ways of expressing the results of the Volume and Heaviness task, it was interesting to explore whether the experimental group improved its performance from the pre- to the post-test, when compared to the control group that did not engage in the CAME activities. The expectation was that the improvement would be greater in the experimental than in the comparison group, in order to support the

intervention as a successful training instance for developing prospective teachers' reasoning skills.

Table 26 presents information regarding the number of participants from the experimental and the comparison group, expressed in totals and percentages, that improved, maintained and decreased their performance in the post-test in contrast with the pre-test. It is possible to observe that, in the case of both measures (the reasoning level and the number of correct responses), a greater number of participants from the experimental group than from the comparison group improved their performance in the post-test in comparison with the pre-test.

**Table 26: Experimental and Comparison Group summary changes in terms of reasoning levels and the number of correct responses**

Performance from Pre- to Post-Test	Reasoning Level				Correct Responses			
	Experimental Group		Comparison Group		Experimental Group		Comparison Group	
	N	%	N	%	N	%	N	%
<b>Improved</b>	16	61.54	8	53.33	18	69.23	6	40
<b>Maintained</b>	8	30.77	5	33.33	4	15.38	3	20
<b>Decreased</b>	2	7.69	2	13.33	4	15.38	6	40
<b>TOTAL</b>	26	100	15	100	26	100	15	100

Even though the raw numbers shown in Table 26 might suggest that the CAME intervention was successful, because a greater number of experimental participants improved both their reasoning levels and the number of correct responses, in order to draw a reliable conclusion it is necessary to further explore if that difference is statistically significant. Thus, the next section will present the statistical tests I conducted with the purpose of exploring this difference.

### *C. Differences between the experimental and the comparison groups in terms of the number of correct responses and reasoning levels at the end of the CAME course*

Given that the reasoning levels expressed in five different categories and the number of correct responses correspond to two different kinds of data (ordinal and ratio respectively), I needed to use two different types of statistical tests.

Firstly, in order to explore if the difference in terms of the number of correct responses between the experimental and the comparison group was statistically significant, I used analysis of variance, or ANOVA, not only because the data that I was going to analyse were ratio data, but also because I intended to compare more than two means. Essentially, ANOVA tries to explain the variation in the data by separating the effect according to different explanatory sources (Cohen et al., 2007). In this particular case, the sources were the effect of time (pre- and post-tests), the effect of the group (experimental and control) and the effect of the interaction of the time by the group.

Consequently, I ran an ANOVA of two factors (time and group) with repeated measures for only one factor (time: reasoning level, pre- and post-test). In other words, the time was the within-subjects factor, which means that the same subjects were measured at two different times (at the beginning and at the end of the intervention) using the same variable (reasoning level). In turn, the between-subjects factor was the group, which means that each subject was assigned to only one of the two conditions (experimental or control group).

Before running an ANOVA, it is important to check that the assumption that the data are normally distributed is being met. In order to explore this, I ran two Tests of Normality of my data, with their corresponding histograms separated by time, one for the number of correct responses during the pre-test (see Appendices P and Q) and one for the numbers of correct responses on the post-test (see Appendices P and Q). Both tests permit me to claim that my data are normally distributed.

After establishing that it is possible to assume that the data are normally distributed, I could move on to the presentation of results from the analysis. Table

27 shows a summary of the descriptive statistics for the experimental and comparison group during the pre- and post-tests.

**Table 27: Descriptive Statistics of the number of correct responses**

		<b>Pre-Test</b>	<b>Post-Test</b>
<b>Control</b>	<b>Mean</b>	9.33	9.33
	<b>Std. Deviation</b>	2.469	2.469
<b>Experimental</b>	<b>Mean</b>	10.08	11.23
	<b>Std. Deviation</b>	2.279	2.268

In this table, it can be seen that the comparison group has exactly the same numbers for the pre- and the post-tests, both in terms of the mean and the standard deviation, which is very unlikely.

Therefore, I re-examined not only my data set, but also the tests themselves in order to double check the information. In the process, I realised that, even though the numbers are correct, they do not necessarily indicate that each person in the comparison group got the same number at Time 1 (pre-test) and at Time 2 (post-test), but that the combination of numbers in both moments is the same and, for this reason, the mean is the same. Based on this information, it is possible to suggest that the reason for obtaining exactly the same numbers for the comparison group might be because there is no real variation in it across time (for a graphic representation of the lack of variation across time, please refer to Figure 3 in Appendix R).

Moving to the experimental group results of the analysis of variance described in Table 27, it is possible to observe that, in contrast to the comparison group, the experimental group increased in terms of the average number of right responses from the pre- to the post-tests, which means that the intervention might have been successful in improving prospective teachers' reasoning levels, as expressed by the number of correct responses.

**Table 28: Tests of between-subject effects**

Source	df	F	Sig.	Partial Eta Squared
Intercept	1	789.483	.000	.953
Group	1	3.446	.071	.081
Error	39			

Having dealt with the multivariate tests results, it is now possible to describe and analyse what these results signify and the implications thereof. Tables 28 and 29 show that there is no statistically significant main effect in terms of differences between groups ( $F = 3.446$ ;  $p > 0.05$ ;  $df = 1$ ), which means that the effect is really driven by the statistically significant ( $F = 4.599$ ;  $p < 0.05$ ;  $df = 1$ ) interaction of time by group (Time \* Group). Thus, time is also a significant main effect ( $F = 4.599$ ;  $p < 0.05$ ;  $df = 1$ ).

**Table 29: Analysis of variance results**

Effect	Df	F	Sig.	Partial Eta Squared
Time	1	4.599 <sup>a</sup>	.038	.105
Time * Group	1	4.599 <sup>a</sup>	.038	.105

In other words, the results of the analysis of variance suggest that, at the end of the intervention, the differences between the means of the experimental and the comparison groups are not explained by the individual differences of each group, but by their differences across time: the intervention. In terms of the implication of these results, it is possible to state that this is what I expected of a successful intervention. In other words, the fact that the time is a significant main effect means that it is significant for both groups. However, as the groups

themselves are not statistically significant, this means that the main effect of time is actually explained by the interaction between time and group.

In order to further explore and to find support for the conclusion that the main effect is actually the interaction between time and group and that this effect is explained by the differences between the pre- and post-tests of the experimental group that were not observed in the control group, I ran two paired-sample t-tests as post hoc tests: one for the experimental group and one for the comparison group.

This analysis showed that the experimental group improved its performance in the post-test when compared to the pre-test. This progress is made clear in the negative value of the mean ( $x = -1.154$ ), which reflects that the average of correct responses at the beginning of the intervention was statistically significant less than was the average of correct responses at the end of thereof ( $t = -3.883$ ;  $p < 0.05$ ;  $df = 25$ ). In contrast, the comparison group maintained its performance from the pre-test to the post-test, which means that the average of correct responses of the comparison group at the beginning of the intervention was not statistically different from the average of correct responses at the end thereof ( $t = 0.000$ ;  $p > 0.05$ ;  $df = 14$ ). For more information about the paired sample test for the comparison and the experimental group, please refer to Appendix S.

These test results initially led to the conclusion expressed in the preceding paragraph, namely that, in the interaction's main effect of time by the group, the differences lie in the experimental group and not in the comparison one. However, for further support, it is necessary to explore if the differences between the experimental and the comparison group at the end of the intervention could be explained by the effect of participation in the CAME course and not by previous differences observed between the two groups.

Therefore, it is necessary to establish that both groups were similar at the beginning of the intervention. In order to ensure that it was possible to attribute the difference between the experimental and the comparison groups to the CAME course and not to other variables present before the intervention, I ran two independent sample t-tests, one for the pre-test and one for the post-test. The results of that analysis confirmed that it is possible to suggest that the differences

between the experimental and the comparison group can be observed at the end of the intervention and not at the beginning thereof.

In fact, the t-tests showed that there were no statistically significant differences between the experimental and the comparison group at the beginning of the intervention programme ( $t = -0.976$ ;  $p > 0.05$ ). Consequently, if it is possible to assume that the experimental and the control groups are similar in terms of the average number of correct responses at the beginning of the term, but they present statistically significant differences ( $t = -2.499$ ;  $p < 0.05$ ) at the end of the term, then those differences could be explained by the effect of the CAME intervention (for more detail, see Appendix T).

Most of the results presented so far have been based on the analysis of ratio data (the number of correct responses) from the Volume and Heaviness Task of the Science Reasoning Task test (Shayer, 1977). Nevertheless, as I described previously, I also had non-parametric data based on the same information but which was coded in a different way. Specifically, the five reasoning level categories that resulted from the application of each of the SRTs were ordered from 1 (lowest reasoning level) to 5 (highest reasoning level), which correspond to ranked or ordinal data. Therefore, I used this second type of data to double check the conclusion that the experimental and the comparison group were similar at the beginning of the intervention, but different by the end thereof. Again, if this difference existed, it could be explained by the participation of the experimental group in the intervention.

In order to analyse this data, I ran two Mann-Whitney U tests that are specifically designed to deal with ordinal data. The first test attempted to explore whether there was a difference, at the beginning of the intervention, between the experimental and the comparison group in terms of the participants' reasoning levels. Based on the mean ranks observed, it is possible to suggest that the experimental group showed higher reasoning levels (Mean Rank = 22.94) than did the comparison group (Mean Rank = 17.63) at the beginning of the intervention (for more detail, see Table 12 in Appendix U). Even though there was an initial difference between the experimental and the comparison groups, the analysis confirmed that which was previously observed in the analysis of variance; namely, that the difference is not statistically significant ( $U = 144.500$ ;  $p > 0.05$ ).

As there was no statistically significant difference between the experimental and the comparison group in terms of their reasoning level at the beginning, this indicates that the experimental and the comparison groups were similar at the beginning of the CAME course. The next step was to run another Mann-Whitney U test in order to explore whether there was a difference between the groups at the end of the intervention. The analysis showed that the experimental group had higher reasoning levels at the end of the CAME course (Mean Rank = 23.79) than did the comparison group (Mean Rank = 16.17). In addition, that which was previously observed in the analysis of variance was confirmed, namely that difference is, on this occasion, statistically significant ( $U = 122.500$ ;  $p < 0.05$ ). For more details, see Table 12 in Appendix U.

As has been observed in this section, the Mann-Whitney U test results support the ANOVA test results in the sense of suggesting that the differences between the experimental and the comparison group, in terms of reasoning levels, might be explained by the participation of the experimental group in the CAME course. This is not surprising, given that the data rely on the same data set that was coded in two different ways. In addition, it is possible to argue that the statistical analysis presented in this section also supports the results in terms of prospective teacher change, as described in earlier sections of this chapter.

### **Evidence from the interviews, learning journals and field notes**

This section will present the findings that arose from the qualitative analysis of the interviews, learning journals and field notes. The results will be presented in a way that takes into account the assumptions and methods that were described in the methodology chapter (for more details, see Chapter V, p. 102). As mentioned in that chapter, this research adopted the interpretative paradigm, which implies the recognition that reality and meaning are socially constructed and are not a given. For this reason, not only were a variety of data collection methods used in order to maximise their explanatory potential, but the information gathered using all these methods were also analysed with the purpose of answering the research questions addressed by this research. In so doing, I was



consistent with my belief that reality is extremely complex and that complexity could be better approached from different points of view and by using different research techniques.

In terms of the methods, this research used a mixed method approach that consisted not only of using different kinds of qualitative methods (interviews, learning journals and field notes), but also quantitative ones (Science Reasoning Task test). However, the qualitative research techniques played a more central role in answering the research questions, and the quantitative test contributed to that process from a secondary position. Based on all these previous considerations, the structure of the following subsections deals, in a clear and straightforward way, with the research questions and objectives that this work answered by considering all the qualitative data collected by the different sources as a whole. In a separate subsection, the Science Reasoning Task test results are presented, and the results of the statistical analysis are integrated with the rest of the data in the discussion section (for more details, see Chapter VII, p. 211).

Within each of the subsections, the results will be organised according to their relative frequency. This means that those findings that were present in more of the prospective teachers will be presented first. In addition, each result will be accompanied by a number expressed in the following terms:  $x/11$ , which means how many prospective teachers of the 11 that were interviewed mentioned the topic. In turn, those topics that had a higher overall frequency will be next. This is because, from my point of view, if a certain topic was mentioned by a prospective teacher (0 = it did not appear, 1 = it appeared), this provides more information than the number of times that the same teacher mentioned the topic, because the latter approach is more sensitive to the coding style of the coder. In this sense, if the coder tends to code shorter pieces of text, this will tend to increase the number of quotations associated with each code during an interview.

It is extremely important to clarify that presenting first the topics that were mentioned by either a higher number of participants or more frequently, considering all the interviews together, is only a way of organising the presentation of themes within each of the result's subsections. For example, there are many codes that are related to the views that prospective teachers had regarding the impact the course had on their reasoning levels, what they think

about thinking skills and the importance of developing them (first subsection of this chapter). Thus, within that subsection, the most frequent codes will be presented first. This way of organising the presentation of results is not a result of the process of analysing the data, but is merely a way of presenting the results for explanatory purposes. When following a content analysis procedure within the framework of an interpretive research approach, all the findings are equally relevant to the researcher. For a detailed description of the coding process and the resultant code structure that gave rise to each of the subsequent subsections, please refer to Chapter V, p. 144.

#### *D. Self-perceived impact of the CAME course on prospective teachers' thinking skills, their views on thinking and the importance of promoting it*

This subsection explores the first two specific research question of this thesis: How do the formal reasoning skills of prospective teachers change after experiencing a course of cognitive acceleration activities? How do prospective teachers' views regarding the importance of thinking change following a cognitive acceleration intervention?

Considering these research questions, it is perhaps not surprising that all the prospective teachers (11/11) mentioned in some way that the course had an impact on their thinking skills, but it is relevant that this finding had the highest frequency ( $f=79$ ) in the study. Prospective primary teachers who reported the impact of the CAME course on their thinking skills could be categorised according to three different perceptions. Firstly, a group of prospective teachers reported a direct impact on their own reasoning ability; in other words, they declared that the activities that we used during the course played a crucial role in the development of more structured and complex thinking patterns. For example, Jessica stated:

*“Well, the workshop has helped me a lot in the sense of giving me certain strategies and knowledge; for example, how to sort different kinds of data. It also*

*gave me good experience of ways of creating different kinds of charts. We often had to find a solution to the problems you posed to us, so it also taught me how to formulate a hypothesis and to think of ways of finding the best solution...before the workshop, I was always asking, 'Do you know what I mean?' I'm not doing that anymore, because I have learned how to structure my own thinking and am able to finally get to a solution"* (Interview, August 2012).

The second group of perceptions refer to prospective teachers who claimed that the activities required the use of their own thinking skills in order to solve them. In that sense, even when the proposed problems were placed in the context of mathematics, it was not sufficient to make use of common algorithms, formulae or procedures, as complex processes were needed in order to arrive at a potential solution. The subsequent statements by Emma are illustrative of the ways in which CAME activities required effort and were thought provoking for prospective teachers:

*"[The course] helped me to...accelerate my thinking, because I really had to think hard to get to an answer...I needed to get to a solution, I needed to understand, to comprehend...I think that's what is missing nowadays is challenging the students...[the course] was a challenge for me"* (Interview, August 2012).

The fact that the course presented challenging activities or problems to prospective teachers is a surprising issue, especially considering that one of the pillars of the cognitive acceleration programme is to promote cognitive challenges. However, as I had not told them this explicitly, they would have had to infer this from taking part in the activities. Clearly, one of my initial concerns when formulating this study was the degree to which the CAME materials could be adapted for a different age group and to a different context from that in which it was originally designed. What I hoped would happen is that the adaptation of the materials would be sufficient to promote thinking and avoid the prospective teachers utilising their current mathematical thinking to solve the set problems. Therefore, if prospective teachers felt that they were being challenged, that could

suggest that at least part of the CAME course outcome was achieved and that the adaptation of the activities from school students to Bachelor of Education students was satisfactory. In other words, this might imply that the problems were sufficiently difficult to move prospective teachers forward in terms of their cognitive development.

Finally, some of the participants claimed that the CAME course was useful in helping them to develop the necessary tools to design lessons with a conscious and deliberate focus on promoting these types of skills in their future students. This perception can be seen in the words of Emily:

*“From a more professional point of view, I feel that many tools were provided that will be important to have as teachers...in other words, I think that the activities are very constructive...I think they have to be guided by the teacher, always making the objectives explicit in terms of ‘what do you want to learn, what do you want to achieve with this activity’ and I think that this could be very useful for my pupils...in order to develop their logical thinking”* (Learning Journal, November 2012).

At the beginning, it was not straightforward to assume that designing a course that tries to promote and develop thinking skills in prospective primary teachers would generate meaningful experiences, both in terms of learning as university students and as future teachers, because this was the first time that this approach was undertaken within an initial teacher training context. However, the findings illustrated by Emily’s quotation provides some evidence that supports my original belief that a CAME course could provide prospective teachers with powerful learning experiences that could be of benefit to them professionally. This point will be further addressed in the section, which deals with prospective teachers’ professional experiences during the CAME course (Chapter VI, p. 186).

Another topic that is relevant because it deals with an important cognitive acceleration pillar is the promotion of metacognition during the sessions. This was described in the literature review, in which I explained that the cognitive acceleration approach intentionally promotes participating students to think and to

reflect on both the individual and the group's thinking processes. This aspect originates from the idea that participants should come to think of themselves as 'thinkers' who have a degree of control over their own thinking process. The ability and the disposition to reflect on how one has solved (or even failed to solve) a problem is a powerful tool that enables children to have more control of their own learning (Adey, 2008), which is why it is a learning outcome that CA programmes try to accomplish. At the same time, if prospective teachers became more competent at talking and thinking about their own thinking processes, they could be more likely to promote cognitive awareness in their own classrooms.

Apparently, the course intention of encouraging prospective teachers' thinking and awareness of their thinking processes was clear to almost half of the interviewed participants (5/11). Those who were aware of this purpose not only talked about this, but also did so in a positive way in the sense of valuing the effort made:

*"...at the end of each session, you always asked us 'how did you do it?' ...and you made us write down the various reasoning skills we had used during the session... in some sense to do the closing of the activity, the metacognition...For me, that was important, because one can solve the problem and that's it, but thinking about how we did it? That was important...in terms of being aware of what was missing..."* (Lucy, Interview, August 2012).

Despite the positive results found in terms of the influence of the CAME course on participants' thinking and metacognitive skills, it was not possible to observe increasing levels of confidence regarding their preparedness to promote this type of ability in their students in the near future. In fact, only five of eleven (5/11) prospective teachers declared that they felt fairly confident about being able to promote reasoning abilities in their classrooms:

*"[I feel] fine. I think I can do it, especially divergent [thinking], it's like a strength I have"* (Jessica, Learning Journal, December 2012).

At the same time, three out of eleven prospective teachers explicitly stated that they were not being sufficiently prepared to achieve this goal in their future classrooms:

*“It's much harder to make someone think and it's also not so easy to determine if that person is really thinking, especially if you have 25 or 40 students in your class”* (Sarah, Learning Journal, September 2012).

Finally, three prospective teachers did not mention any feelings or thoughts regarding this topic.

Related to the issue that prospective teachers did not feel more confident about promoting thinking skills in their students is the fact that it was not possible to observe an increment in their views regarding the importance of encouraging students' thinking. I am not able to state that they did not change their initial views regarding the importance of encouraging thinking at the end of the intervention, because they simply did not report their ideas concerning this subject explicitly. In retrospect, I should perhaps have questioned them about this but, in the interviews, my aim was to allow the ideas and issues come from the participants rather than by prompting them to discuss specific topics. This is a compromise that the researcher has to make when conducting semi-structured interviews.

There were only two (2/11) prospective teachers who mentioned this topic and one of them stated:

*“I think that [thinking] is crucial. I feel that you don't learn if you're not thinking about the content you're learning. Often, the content is presented and you can memorise it, but then it's gone. When you really think, that knowledge is long lasting”* (Amy, Interview, August 2012).

Even though most prospective teachers did not explicitly mention how and why it is important, from their point of view, to encourage students to think inside

their classrooms, they did so implicitly (8/11,  $f=18$ ) by criticising the current situation in which most schools and/or teachers do not encourage students to think:

*“Usually, and regrettably this is found in many schools, students don't think and they only learn how to follow instructions. (...) I think it is crucial that students learn how to think by themselves. (...) This might happen in some schools, but not in most of them”* (Jessica, Interview, December 2012).

The lack of confidence regarding their competence in preparing activities that promote thinking skills in their students and the absence of explicit recognition of how important these skills are inside the classroom might be associated with the fact that most of the prospective teachers (7/11,  $f=8$ ) were not able to give a coherent explanation when they were initially asked for a description of thinking skills, even when I told them that they did not need to use sophisticated terms and should use their own words to explain the concept. The difficulty in articulating a definition of thinking skills was openly admitted by prospective teachers and is reflected in quotations such as the following:

*“I didn't have the concept of thinking skills in my mind”* (Amelie, Learning Journal, August 2012)

*“I think that [thinking] is to take the knowledge to a more abstract or more concrete level...I really don't know...”* (Molly, Interview, August 2012).

The aforementioned lack of knowledge, in conjunction with naïve views about thinking, was reflected in the descriptions the participants gave when they were asked about thinking skills. Five prospective teachers (5/11) defined thinking as a type of common sense skill that one uses to behave in a specific context:

*“It’s...the way I formulate and organise the things that are inside my head to communicate, to interact with my environment and with the people around me”* (Emma, Interview, December 2012).

Another group of participants (6/11) held views about thinking that were almost synonymous with mind or, in other words, everything that takes place inside one’s head:

*“Everything that is going on in my mind...for me, that's thinking”* (Lucy, Learning Journal, August 2012).

Finally, the last view expressed about thinking was the amount of knowledge that students have:

*“To me, intelligence depends on knowledge; so, if you have less knowledge, you're thinking less and you have fewer abilities”* (Olivia, Interview, August 2012).

Based on the kind of arguments they used and the views they described, it is possible to imagine that these could have been developed in informal contexts as a result of general and common knowledge that is accessible in their everyday lives. In addition, the fact that most of these quotations came up during the follow up interviews might suggest that the CAME course raised the issue and the course could have appeared to be a prototype of what a class that encourages thinking looks like. Furthermore, as the course was quite different from those they had experienced in a regular school or University classroom, this might have led them to conclude that thinking is not being sufficiently emphasised inside the classroom. I will come back to this hypothesis later in another section that provides further discussion regarding this issue.



*E. Prospective teachers' change in terms of their views  
regarding teaching and learning in general and about  
mathematics in particular*

The following subsection deals with the third research question of this thesis, 'How do the views of prospective teachers towards teaching and learning change following a cognitive acceleration intervention programme?' In other words, whether it were possible to observe a difference between the views that prospective teachers held regarding teaching and learning before and after the course.

During the interviews I conducted at the beginning of the course, I asked the participants if they could describe the way in which children learn by drawing on what they had learned at university and from their experiences in the classroom. The most frequently observed answer to this was that children learn in a concrete way. This view was not only mentioned by all the interviewees (11/11), but also had a high relative frequency ( $f=26$ ). It would seem that the students meant two different yet related things by the term 'concrete'.

In the first place, by 'concrete' they meant that the learning process should start with the manipulation of something familiar to their students in order to attract their attention. This could be appreciated in the following quotation:

*"I think that the best way is to work with concrete materials with your students, apart from the blackboard, of course. For example, we have a teacher who always tells us that we need to work at three different levels: concrete, graphic and symbolic, and that we always have to start from the concrete level. What I mean is that students always have to have something to manipulate. For example, today I was being taught how to use the abacus to teach addition and subtraction. I think that they [the teachers] are right...I think that students learn more easily if they are manipulating some kind of material than if they are just listening" (Zoe, Interview, August 2012).*

The second meaning they assigned to the term concrete was that students' learning experiences should always be connected to their everyday life experiences. In other words, the acquisition of each new concept must be connected with the children's previous knowledge or must be in line with the students' own reality, experience or context. For this reason, Lucy states:

*“[Children learn] by giving them the context of the new knowledge because, if you start explaining something that doesn't make sense to them, or if it isn't familiar to them, they won't learn it. During my internship, I've seen that some things that are really easy and basic for me, and which are really easy to learn, couldn't be learnt by the children because they weren't told what the information was and it was also not put into context”* (Interview, December 2012).

Prospective teachers described not only the learning process in general as being concrete, but also the process of learning mathematics in particular (4/11). The words of Sophia on this topic are as follows:

*“...it is necessary that children manipulate the concrete material, that the teacher stands up in front of them and solves the problem with them...students need to observe nature, they need to explore, that's what works best in the end. I think that, with mathematics...the thing that I love is the concrete material. However, it's essential that children make different kinds of things with whatever they have to hand”* (Interview, August 2012).

According to the interviewees, the advantages of promoting concrete learning experiences inside the classroom are

- i. that students learn faster: *“...when you work with more concrete things, it is much easier to learn”* (Olivia, Interview, September 2012)
- ii. that learning is more meaningful: *“How do they learn better? With didactic, concrete material, children learn by doing, not by sitting and listening as passive students...In other words, it's very different to be*

*standing in front of your students talking and talking, and the child may be listening to you, but s/he is not achieving meaningful learning. S/he will forget the information the next day but, if the child manipulates something, the answer will come from him or her, which will be much more meaningful”* (Emma, Learning Journal, August 2012).

The second most frequent view about learning, which is closely related to the previous one, is that students should play an active role in their learning processes by discovering and/or constructing their own learning (6/11,  $f=10$ ). It was very clear in the discourse that the students thought that pupils do not learn, at least not meaningfully, if they are sitting quietly at their desks and listening to what the teacher is telling them. These kinds of statements are coherent with what constructivist theories claim about the process of learning, which might suggest that prospective teachers held constructivist views about teaching and learning. Accordingly, prospective teachers claimed that students’ learning takes place when they are actively involved in and engaging with their own learning processes:

*“...discovery means that...you always have to look at ways in which children can infer their own learning and make discoveries...I don't know...to discover the meaning of words, learning has to be active for children, they are constantly looking for ways to solve things...”* (Emily, Interview, December 2012).

The third and last learning view that was expressed at the beginning of the course was the role that emotions play in the learning process (4/11,  $f=12$ ). In this sense, prospective teachers claimed that they should create a safe emotional climate in their classrooms in order to facilitate their students’ learning because, if students do not have a positive emotional disposition to learn, then learning would be difficult to achieve:

*“I think that, as a teacher, you have to be dedicated to your students even if you have a big class. You have to be dedicated...you have to be aware of the difficulties of each student, what they lack...because sometimes their difficulties are related to their families, they have problems at home and that's why they can't concentrate during your classes. I think that if you care about your students, you'll develop good methodologies in order to promote their learning (Zoe, Learning Journal, October 2012)”.*

The views about teaching that the interviewees shared at the beginning of the course did not have frequencies as high as the ones related to learning, but they did reflect similar views about teaching and learning. Firstly, they stated that teachers should try to identify their students' learning needs and strengths in order to be able to improve their learning experience (4/11):

*“Well, I think that teachers always have to be attentive to their students, to see and to identify the students who have special learning needs, and to provide extra support for them...” (Emily, Learning Journal, September 2012).*

Prospective teachers identified the role that teachers should play in their students' learning as that of a mediator (2/11). They did not go into detail in the sense of explaining what they understood by mediation, but they did refer to this point explicitly by using that term:

*“As a teacher, you have to be the mediator of your students' learning. You have to be there to pose them a problem, to teach them the alphabet, to teach them how to add, to go along with them during their processes, but they are the ones who have to discover the final result” (Emma, Learning Journal, September 2012).*

Even though they claimed that students should play an active role in their learning processes by discovering and/or constructing their own learning and that teachers should mediate the learning process of their students, what they said

about ways in which teachers can support learners did not seem to be directly linked to what they said about learning. It is possible that the mediator role is a better fit with the socio-constructivist approach. However, the kinds of examples they gave (to teach the alphabet, how to sum and so on) does not sound much like problem-solving/discovery-type learning. This could suggest that their ideas are not yet fully formed; thus, even though they know what is expected of them and what they would like to do inside the classroom, they might not yet have experienced how this is achieved in a classroom situation.

These contradictions were present in the discourse of various prospective teachers and will be further explored in the next paragraphs. My own impression as a researcher is that, although they have heard and learned some constructivist and socio-constructivist theories about teaching and learning, they have not seen them in practice during their experiences at school or as undergraduate students. Therefore, even though they are able to talk in a constructivist or socio-constructivist manner and to understand the advantages of that approach for teaching and learning, they do not have a coherent mental image of how a teacher actually puts these principles into practice or of good examples of this kind of practice.

Finally, two prospective teachers also claimed that one of the most important things in the teaching process are teachers' expectations about students' learning (2/11). In other words, if teachers do not believe in their students' capabilities and do not have high expectations of their ability to learn, this might prevent students from learning. This perception can be appreciated in quotations like the following:

*“Based on my internships, I've learned that the most important thing is what teachers expect from their students. Some teachers have told me, “Don't waste your time with him, he's not able to do it”. In contrast, if the teacher is hopeful about a student, that student will be able to move on”* (Molly, Learning Journal, October 2012).

As I said before, even though these prospective teachers' views about teaching and learning sounded congruent with the latest educational theories, most of the interviewees (6/11,  $f=7$ ) made comments that led me to infer that they held contradictory theories in this regard. For example, Olivia declared:

*(...) it's been said that pedagogy has to be constructivist, as do teaching and learning, but there also have to be a degree of behaviourism, because the teacher must provide a foundation for the content...In the end I feel that the teacher has to ask for silence in the classroom in order to be able to create a space for delivering the content and, from then, on the students could construct their own knowledge (Interview, December 2012).*

Based on this quotation, it might be suggested that the interviewee does not sound completely convinced about the view of teaching she starts to describe at the beginning of the quote. By the end of the quote, it seems that she thinks the first stage of the teaching process should be centred on the transmission of content from the teacher to the students in order to give them a certain knowledge base and, only after that, would a more constructivist stage in which the students have the space to construct their own knowledge take place. As I said before, most of the interviewees (6/11) reflected simultaneous and contradictory views about teaching. This phenomenon might be related to two different factors:

- i. the limited professional practice (or internships) that prospective teachers have had so far during their initial teacher training, meaning that their discourse about constructivism could be mainly theoretically grounded
- ii. the lack of constructivist teaching models, both at school and at university, that could have given them the chance to observe constructivist principles in practice.

These hypotheses are, to some extent, supported by the comments participants made when the CAME course was complete. Firstly, they started to strongly criticise how mathematics is usually taught in Chilean schools (9/11,

$f=46$ ), in the sense that processes and mistakes receive less emphasis than do the results themselves; thus, learning and understanding the reasons behind mathematics do not receive focus. Interviewees also mentioned that teachers frequently do not allow the development of different processes to get to the same result, but demand that students follow exactly the same algorithm that was shown during the class. The other criticism they mentioned is that mathematics is usually taught in a mechanical/theoretical way that does not promote thinking, understanding, abilities, application and/or transference.

The aforementioned criticism is reflected in quotations such as the following:

*“Many times, what they teach us in mathematics is a list of contents and some formulas, which is presented as being the only way you can solve certain problems...for example, what you most frequently see promoted in the [mathematics] classroom is to try to solve a problem by using a formula or an algorithm (...) but what about the analysis, what about the evaluation of different methods? That's not often promoted in the classroom, the most common approach is to try to apply a formula, and that's it”* (Zoe, Learning Journal, November 2012).

Only 13 of the 46 quotations that criticise the way in which mathematics is currently taught were mentioned during the pre-course interviews, which means that the other 33 arose after the course was over. This is one of the reasons that I think that it was a new experience for them to see socio/constructivist principles put into practice. After participating in the course, they could have had the opportunity to contrast what they said about teaching mathematics and what they experienced regarding teaching and learning mathematics with what they experienced during the CAME course. While this process may have caused them to say more about their beliefs regarding teaching and learning mathematics, the issues raised indicate that they formed a new or adapted version of how mathematics might and could be taught.

In fact, even when prospective teachers' experiences during the CAME course could have been similar to what they had heard about how teaching mathematics should be, at the end of the course most of them (9/11,  $f=29$ ) reported that it was very different from what they had seen regarding teaching and learning mathematics. Specifically, they claimed that the methodology we used during the course was new for them and was very different from what they were used to in other math courses at university or at school, and that it was also a novelty for them to talk about the relevance of developing thinking skills in their students:

*"...they [the Department of Education] did not tell us what this course was going to be like but, when you presented the first session and the activity, I found it novel and different. (...) it was something we weren't used to. From the first activity, we realised that this class was going in a different direction, that it wasn't similar to the other [courses], that we were going to learn something meaningful, that we're going to benefit from this workshop, from this knowledge and from these group experiences. So, in the end, we came for those reasons"* (Zoe, Interview, December 2012).

In the same vein, during the follow-up interview, Jessica commented:

*"In the classroom, teachers usually tell students what to do and how to do it, with instructions, with everything...I mean...it's very...very structured, and they don't usually let students express themselves, they are not encouraged to think at all, they have everything done for them...I really think that this course has been completely different from all the other courses I've attended because, in mathematics, you are usually exposed to content, but not to content that is related to such concrete things (...) there were two occasions when our maths teacher [at university] gave us a problem and we had to find a solution for it, but none of the problems were meaningful, we just had to apply some formulas to solve them"* (Interview, September 2012).



The findings that were presented in this subsection show how prospective teachers claim they changed their views regarding teaching and learning in general and about mathematics in particular after participating in the CAME course. In this context, it is plausible to think that they moved to a more constructivist stance and expressed fewer contradictory views about teaching and learning. This is not only because they experienced the constructivist principles put into practice as students, but also because they were consciously aware of it and talked about this explicitly at the end of the course (7/11,  $f=15$ ):

*“I think that the teacher's role is also important. This University has a plan and a mission to develop constructivist teachers, but I haven't seen this in every course...but this course makes a contribution to the constructivist training that the University is looking for in order to create constructivist teachers...you presented us with a model, because teaching math is very complicated, I personally think so...I think that you broke the mould of the close-minded math teacher who only cares about the result and not about the procedure (...) Yes, I think you're a model for us, a model that we can follow when we teach”* (Olivia, Learning Journal, October 2012).

#### *F. The influence of the cognitive acceleration approach on prospective teachers' attitudes regarding teaching and learning in general and about mathematics in particular*

During the interviews, most prospective teachers (9/11,  $f=22$ ) described their bad experiences with mathematics either at school or at university. In many of the cases, these unpleasant memories associated with learning mathematics were linked to the teachers that they had during their trajectories as mathematics students/learners. In this context, the difference between the number of quotations that related to bad experiences when learning mathematics ( $f=22$ ) versus the quotations connected to good experiences with it ( $f=6$ ) is very interesting.

Some of the bad experiences prospective teachers shared with me are the following:

*“I think that [I have hated maths] since I was a child. I had bad experiences with my math teachers. They always...I always found it difficult, and teachers never took me into consideration, they didn't explain things to me well. So, because they didn't explain things to me, I felt silly...I didn't like it and I simply blocked maths out and didn't want to learn...I was in second grade and the teacher gave me an E, even though I really put in a lot of effort. All my classmates got an A or a B. I remember that the teacher suggested that I was a bit stupid when it came to maths. That was when I put maths on my blacklist and I have failed maths ever since. I think these are the kinds of bad experiences children have during their childhood” (Jessica, Learning Journal, November 2012).*

Similarly, Sarah commented:

*“I fought...from first to twelve grade with my maths teacher. I couldn't understand what maths was for. I thought that calculators could do all the work, which reflected my views about maths. Why do I have to do it if the calculator can get to the result? Do you know what I mean? ...and when you told us in the first class that the workshop was going to be related to mathematics, I was immediately discouraged. I said, 'Ah! How boring!' So my expectations about the course were negative” (Interview, August 2012).*

The predominance of bad over good experiences related to learning mathematics might be associated with prospective teachers' lack of confidence regarding teaching mathematics. During the pre-course interviews, seven of the eleven participants mentioned in various ways that they did not feel sufficiently prepared to teach mathematics (7/11,  $f=23$ ), which may be related to the absence of solid and positive models of good mathematics teaching. The difference between the number of quotations related to being confident ( $f=9$ ) and not confident ( $f=23$ ) regarding teaching mathematics is very similar to the difference between good and bad experiences with learning maths mentioned in the previous paragraph.

The previously mentioned lack of confidence about teaching mathematics is reflected in the following quotation:

*“My weakness is mathematics...I'm becoming reconciled to this, because I always was bad at maths. At school, I failed maths every year when I was in secondary school...but I do feel that it's important because, as a primary teacher, you have to teach every subject and if I don't know how to explain mathematics, even though it'll be my responsibility. If I'm not explaining it well, I'll be responsible if the student doesn't learn”* (Olivia, Learning Journal, August 2012).

Likewise, Lucy reported that she felt

*“...nervous, a bit anxious (...) because I'm not good at maths”* (Learning Journal, September 2012).

As can be seen, the findings of this thesis are consistent with previous literature and research on this topic, which showed how primary teachers experience anxiety and negative attitudes towards mathematics (Ernest, 1989, Smith, 1996, Murphy, 2006, Hodgen and Askew, 2007, Burgess and Mayes, 2008, Henderson and Rodrigues, 2008).

The comments prospective teachers made regarding how confident they felt regarding their preparation to teach in general were not as frequent as their comments about teaching mathematics in particular. However, in the case of their confidence to teach, the situation was more balanced because almost half of the teachers reported not feeling confident about teaching in general (6/11,  $f=7$ ), while five out of eleven declared they felt confident about it (5/11,  $f=6$ ). These two findings can be seen in the following quotations:

*“I feel...a bit scared, because you realise that you don't know everything and maybe you don't know anything. So I feel afraid of facing students who might catch me out”* (Sophia, Learning Journal, August 2012).

*“[I feel] fine, because one of my strengths is my creativity; so, if I see that my methodology is not working, I can change it easily. Therefore, I feel that I'll be able to teach them”* (Jessica, Learning Journal, December 2012).

In the context of the lack of confidence, especially regarding teaching mathematics, it is relevant to remark that most of the prospective teachers (7/11,  $f=20$ ) commented that the CAME course had a positive impact on their confidence about their ability to teach mathematics and to share their reasoning with their peers. With regard to their confidence about teaching mathematics, Zoe commented:

*I have good grades in mathematics...I can't say that my achievement is poor, because I've obtained good grades in my maths courses and in my didactic courses, but I didn't feel confident about teaching maths because I felt I only knew concepts, only theoretical things, just the formulas...so I didn't know an adequate methodology to teach maths. I felt that I was going to teach in the same way in which I was taught: only concepts, formulas and nothing else. I didn't feel prepared, but after this workshop...we hadn't had anything similar to this course, and it helped me to realise how we can develop our abilities to work with our students, to know how can we promote their skills to the maximum. That's why I say that the methodology we used in this course can be used in our classrooms and I think that this is the best method to follow* (Learning Journal, November 2012).

In turn, the next quotations are illustrative of the words that prospective teachers used to describe that they felt more confident about sharing their reasoning with their peers after the course:

*“The truth is that I don’t feel completely confident about mathematics, but in comparison with how I felt before the course, I do feel more confident now...In fact, before the course I would never have gone to the blackboard to explain what I did because I was very insecure. I was afraid of doing it incorrectly, but not anymore...Now, I’m even motivated to look for strategies to teach my students, because I know this will help them, and it is also fun”* (Jessica, Learning Journal, November 2012).

This finding, in conjunction with the fact that prospective teachers began to be more aware of their thinking processes after the course, and were also able to talk about them, could have a positive impact on the kind of activities they promote, the amount of discussion and the culture of thinking they emphasise inside the classroom. This point will be further dealt with in the Discussion chapter (see Chapter VII, p. 211).

In a similar manner, prospective teachers’ confidence in their mathematical capabilities improved. For example, Olivia reported:

*“I feel that I learned a lot. Before the course, I wasn’t able to stand up in front of my classmates to explain something. At the beginning I was ashamed, but during the course I discovered that I do have the tools to get to a result...so I didn’t care anymore if what I was doing was right or wrong, because what I was explaining could help my classmates to understand other ways of solving the problem...because, at the end, we always shared our procedures and compared them...yes, sharing and comparing was important”*. (Learning Journal, December 2012)

The fact that the CAME course led prospective teachers to feel more confident to teach mathematics and to share their reasoning, procedures and results with their peers is a finding that one could expect from a course that uses the cognitive acceleration approach. Every CA classroom promotes a safe climate and a culture of discussion in which all points of view are valued, which is not

particularly common in regular classrooms, particularly in mathematics classrooms, as the prospective teachers described. Given the particular characteristics of the CA approach, it is fundamental that teachers manage to develop a safe classroom climate characterised by respect and collaboration.

Only in that type of atmosphere could every student feel comfortable and confident in engaging with the cognitive challenges that the teacher poses to him or her. However, according to the reports by the participants, this kind of climate was uncommon in their classrooms, especially in mathematics classrooms, where the teachers usually categorised students' answers as right or wrong and usually accepted only one type of procedure to solve a specific kind of problem. For this reason, learning mathematics in a safe classroom environment could have been a novel experience for prospective teachers and might be related to the increase in their confidence levels.

The other attitudinal change that could be observed in prospective teachers after the course was that most of them (9/11,  $f=22$ ) claimed that they had fun during CAME lessons, the activities were motivating and that they could use these types of activities to motivate their students in the future. Emma describes how the CAME activities were fun, saying:

*"...apart from that, the most fun classes use activities that promote discussion inside the class, like 'hey! I think this' and, at the end, everyone has to support what they did...All that makes you keep thinking...'hey! I can do this, I can do that'. It keeps you awake all the time..."* (Learning Journal, October 2012).

Regarding prospective teachers' motivation during the CAME lessons, Emily stated:

*"The activities weren't like regular activities, they were much more... innovative and they also encouraged us to look for a solution, to think hard, and they also promoted our interest in getting to a result (...) In addition, as we were always working with everyday problems, this worked as a hook, it motivated you"* (Interview, August 2012).

Finally, Jessica describes why the activities could be used to motivate her students in her future professional practice:

*“For example, playing bingo and things like that are very useful strategies for teaching certain things to children, apart from being fun for them...that learning will last longer than if you just teach them theoretically, because the learning is concrete. Also, if they are having fun, they’ll be paying much more attention”*  
(Learning Journal, December 2012).

It is worth noting that, given the prospective teachers’ unfortunate histories of learning mathematics, I considered this motivation to be an attitudinal change, because most of them were not initially motivated regarding the course when they discovered that it was related to mathematics.

#### *G. Prospective teachers’ learning and professional experiences during the CAME course*

This subsection deals with the last two research questions, namely how prospective teachers view a cognitive acceleration approach in terms of experiencing the activities as learners and how prospective teachers view a cognitive acceleration approach in terms of the application of the activities to teaching and learning mathematics. These two questions attempt to separate prospective teachers’ learning experiences as mathematics students during the CAME course from their professional experiences as future teachers. Although I am aware that this distinction is artificial, I think it is interesting to explore both phenomena separately in order to better understand how the participants experienced the course as students, as well as how the course could be helpful for them as teachers.

I will begin by describing the experiences that prospective teachers had as students during the CAME course. Many of the findings that will be presented in

this section were explained in previous sections, but the emphasis will now be on their learning experiences. Before starting with a more detailed description of what the course meant for my participants, I think it is worth illustrating the overall experience that prospective teachers had during the course, because that whole picture is frequently lost in the fragmentation of their words.

Because of my dual role as researcher and teacher during the course, my view was that the course was a pleasant and positive experience for prospective teachers. As was stated earlier, most of them (with only a few exceptions) did not like mathematics, because they had unpleasant previous experiences at school or at university. These bad experiences were usually associated with their maths teachers being very rigid in the way that they taught and evaluated students. However, this was not an impediment for them to attend the course, even though it was voluntary. This is reflected in the high level of attendance I experienced during the course, with those who opted out usually doing so after taking the Science Reasoning Task test, which means that most of the students who abandoned the course did not have the experience of even a single CAME lesson. Therefore, those who experienced CAME activities continued to attend the course of their own volition.

In addition, it was possible during the sessions to observe that students were motivated by the activities, which produced high levels of participation and discussion. At the same time, it was obvious that the methodology was new and unfamiliar for them, not only because, at the beginning of the course, they did not know how to react to the problems I posed to them, but also because while I was monitoring their progress during the sessions and asked them '*What are you doing?*', they usually thought I was asking them because what they were doing was wrong. This reflects the traditional approach to mathematics: the answers are either right or wrong, and if the teacher is asking a student something is probably because s/he is doing something wrong. Nevertheless, as time went by, they became used to the method and realised that what I was trying to do was to encourage them to talk about and to reflect on their procedures and reasoning processes and not to judge the procedures they were developing.

The impressions I had during the sessions and which are reflected in my field notes were consistent not only with the experiences that prospective teachers



shared with me during the interviews and learning journals, but also with the final discussion we developed in the last session of the CAME course. This involved the entire class and not certain students, as had been the case with the interviews and the learning journals. It was interesting to note the ways in which their views regarding participation in the course had changed. At the beginning, because they knew the course was part of my doctoral research, they felt that they were helping me because I needed a group of participants. However, at the end of the course, they were the ones who were thankful for having had the opportunity to participate in a course that made such a significant contribution to their professional development. They felt that they had been fortunate, because they knew this course was not going to be offered again the following term.

I will now proceed to a more detailed account of my participants' learning experiences. Prospective teachers described the activities as being sufficiently challenging, which means that they were not so difficult as to impede their engagement during the sessions, but neither were they too easy, they were designed to demand higher cognitive processes. Therefore, the prospective teachers stated that they needed to use their thinking skills as a tool to solve the CAME activities (11/11,  $f=79$ ), which resulted in an improvement of their previous skills. With regard to the usefulness of the course in terms of improving her thinking skills, Amy stated:

*...[the course] helped me to better understand my reasoning from a mathematical point of view...I used to apply memorised formulas and algorithms. In other words, I usually tried to repeat the formulas and algorithms in every context. Therefore, the course helped me to understand my reasoning better. Participating in these workshops made me question all the previously learned formulas and to realise how I think while I'm trying to solve a problem. It helped me to be aware of how I organise my mathematical thinking...so, in the end, that questioning taught me that maths is not a group of unrelated contents, but is a way of reasoning. It taught me to think...I learned to place more emphasis on the process, to question more and to improve my ability to analyse and to reflect (Interview, December 2012).*

The fact that prospective teachers felt that the course helped them to enhance their thinking skills is closely related to their statements regarding the ways in which the activities and the methodology forced them to be more attentive and more conscious of their thinking processes (4/11,  $f=11$ ). As Sarah commented:

*“It [the course] helped me...with my metacognition...in some sense, you become conscious of the process you’re developing. For example, as I told you, if I saw that someone was taking the wrong path and I also had thought that at the beginning but I ruled it out after a while, I thought, ‘Ah! He or she arrived at that level of my reasoning process, but I then moved forward...I became conscious of the processes I performed, I internalised them”* (Interview, December 2012).

It is relevant that, after the course, some prospective teachers were able to verbalise the impact the course had on them in terms of their thinking skills and their metacognitive processes. This is not only because this was one of the intended aims of the CAME course, but also because it could give them the tools to use the same strategies that I had used with them with their students in the future. This is also reflected in the fact that, even though I had never told them that each session had a clear structure and methodology, they began to realise that we followed the same steps during every session, and they valued this structure because it clarified what they were to do and what was expected of them during each of the steps.

In terms of learning mathematics specifically, it was encouraging for me to discover that many of them (9/11,  $f=31$ ) became reconciled to the process of learning mathematics, which most had previously described as being very traumatic. Even though most of them shared negative previous experiences of learning mathematics, the personal experiences they underwent during the course varied. Some claimed that the course helped them to realise that, in contrast to what they had believed almost all their lives, they were good at mathematics or had maths skills. Others reported that the CAME course helped them to improve

their comprehension of the subject which, in turn, enhanced their achievement in other math-related courses.

In this context, it is not surprising that most of the participants (7/11,  $f=20$ ) claimed that the CAME course enabled them to feel more confident about their abilities and/or knowledge and to share their reasoning processes with their peers. As I described previously, many of them had experienced mathematics classrooms in which the culture was intimidating and unsupportive. In addition, the approach taken was the cause of the students' doubting their mathematical capabilities. In contrast, the philosophy and theory behind every cognitive acceleration session demands that the teacher creates a positive classroom climate in which the students feel free and safe to explore and engage in the cognitive challenges that are presented to them, and in which the processes and endeavours are more valued than is merely arriving at the correct answer.

In this sense, the CAME classroom values the process of solving a problem and the act of reflection about that process as much as it does the result itself. It would seem that this was apparent to the prospective teachers, because most of them not only commented that the methodology used during the course was new and unfamiliar to them (9/11,  $f=29$ ), but also that one of the things they liked the most (9/11,  $f=29$ ) was the emphasis on a flexible approach to problem-solving, giving the prospective teachers the opportunity to solve problems in many different ways or from different points of view. This appreciation should not be underestimated, particularly considering the prevalence of the criticism of the mechanical way in which mathematics is usually taught in 'normal' classrooms (9/11,  $f=46$ ). The following illustrates why Olivia values the flexible approach to learning mathematics that characterises CAME lessons:

*...you taught us that we didn't have to follow a particular method that was necessarily the right one to get to the answer. By contrast, you let us find different strategies and, from the strategies we found, to choose the best one to get to an answer for that particular mathematical problem...what had always happened to me previously was that teachers wanted me to answer exactly the same way as in*

*which they had presented the problem, and not with your own ideas*  
(Learning Journal, October 2012).

Part of the reason for the prospective teachers' positive reception of the CAME lessons might have been related to the fact that most of them found the activities to be entertaining and motivating (9/11,  $f=22$ ). In this sense, it could be suggested that they enjoyed the course not only because they thought they were benefitting from it, but also because they had fun during the lessons. As Amelie stated:

*"I felt motivated to go to the workshop, I awaited every Tuesday anxiously. It was almost like a relaxing time for me...to think, to opine, to talk. It was fun, I liked it a lot and I enjoyed it very much"* (Interview, August 2012).

This may reflect that the activities were successfully adapted from the school version to the 'university' version, which was one of my concerns when planning to adapt and use an intervention programme that previously been used exclusively with school students for use with undergraduates.

Despite all the positive learning experiences that prospective teachers reported regarding the CAME course, there were two aspects that, from their point of view, could have enhanced their learning processes. Firstly, in one of the three participating universities, half of the participating students (2/4,  $f=3$ ) claimed that the course could be improved by having a bigger group in order to allow more classmates to contribute to the discussion. I have to admit that I agree with them, not only because it was more difficult to work with such a small group of students ( $n=4$ ), but mainly because their learning experience would have been much more potent if more students could have shared their thoughts and procedures with the rest of the class. This suggests a strong socio-constructivist approach to learning, in which knowledge is co-constructed and then shared or reinterpreted by each of the participants. In the case of this university, as there were only four participants, they were forced to work in two pairs, which did not allow them to work with many different people during each session. Thus, they

did not have the opportunity to listen to a range of different approaches to the same problem. However, it is difficult to estimate the impact this small group work could have had on their learning experiences in comparison to the experiences of the participants from the other two universities.

The second issue was more frequent (4/11,  $f=11$ ) than was the previous, and was raised by participants from all three universities. Some prospective teachers claimed that they would have benefited more from the course if they had had longer lessons every week, or a greater number of them during the term:

*“...I often had the feeling that we were going very fast, we had to solve the problems fast, because many classmates had to leave on time because they had other courses afterwards....so I think it's important that the course should have had two hours [per week] and not only one”* (Emily, Learning Journal, October 2012).

I agree with this point, but only to a certain extent. Each CAME lesson lasted between 50-60 minutes, which is actually the usual duration of any cognitive acceleration lesson. I had the impression that the time was sufficient to fulfil the learning objectives set for each specific lesson. In other words, there was enough time to introduce the activity, present the challenge, give students the time to solve it, discuss what they did, and reflect on how they did it and to try to find new contexts in which those strategies could be applied. However, it is important to remember that this course was not a normal CAME course, but was specially adapted for prospective teachers. In that sense, it had the simultaneous objective of improving their thinking skills and, at the same time, equipping them with various tools that could assist them in their future practice as primary teachers.

However, the second aim was not completely fulfilled within the timeframe of the course. We did not have enough time devoted to considering the transference of the skills from one learning context to a hypothetical teaching situation. However, this issue is more closely related to the next part of this subsection in which I will discuss the aforementioned question, ‘How do

prospective teachers view a cognitive acceleration approach in terms of the application of the activities to teaching and learning mathematics?’

From a professional point of view, the most frequent finding was that prospective teachers claimed that the course helped them to develop certain tools that will allow them to design and focus their lessons in order to develop thinking skills in the future (11/11,  $f=79$ ):

*“[The course] helped me to value the importance of developing thinking skills, to teach thinking. It helped me to realise that, although students may have good grades, they don’t have common sense...in the end, it’s the teachers’ responsibility to teach that. I mean, you can’t be satisfied by only teaching content, you also have to promote your students’ thinking potential...because, in the end, having good thinking skills will help them in their everyday lives and will allow them to learn more”* (Amy, Learning Journal, October 2012).

The fact that prospective teachers felt that they improved their ability to teach thinking skills and their understanding of why this is important is a significant finding since, at the beginning of the CAME course, some of the prospective teachers explicitly admitted that they knew very little about what constitutes thinking skills and how these could be promoted in the classroom (5/11,  $f=8$ ). This lack of knowledge is consistent with previous research and with the literature, which has shown not only that teachers do not have the necessary knowledge or skills to promote these kinds of abilities in their students, but also that these are not currently being sufficiently emphasised in teachers’ preparatory courses (Leat, 1995, Lee, 2005, Barak and Dori, 2009, McDonald, 2010). In this context, it is possible to understand why prospective teachers found that the methodology used in the course was novel and why the topic was new for them, since this was the first time during the Bachelor of Education program that they dealt directly with the issue of promoting thinking skills.

Although these results are encouraging, the course alone is not sufficient to fulfil prospective teachers’ training needs in relation to the development of their future students’ thinking skills. For most of the prospective teachers, this

was their first professional approach to the topic, and the time we had for the course turned out to be very limited, especially considering the range of ambitious objectives I had set at the beginning. This limited time frame might be associated with the reason that only five of the eleven interviewees reported feeling confident about their teaching thinking skills (5/11,  $f=5$ ) and another three explicitly declaring that they did not feel confident about them (3/11,  $f=8$ ).

In the original cognitive acceleration programmes at King's College London, the interventions were designed as 30 sessions delivered once a fortnight over a two-year period. The professional development (PD) programme that was offered to the teachers implementing the course also lasted two years. Thus, as the PD programme was delivered to qualified teachers, they did not have the same competing priorities as the prospective Chilean teachers had. In contrast, only 12 sessions were delivered once a week in this study. The reason for the length of the original CA intervention and of the PD programme is that cognitive development and teacher change takes time. Even though I was aware of this, my hypothesis was that prospective teachers involvement in a CA course could have a positive influence on their training as future teachers, which is why I wanted to explore this aspect. Some of the findings of this research do point in that direction, although the expected impact does have certain limitations that will be explored in depth in a subsequent section (see Chapter VIII, p. 231).

Along similar lines, a small group of prospective teachers (3/11,  $f=3$ ) commented that their professional experience during the course could have been enhanced by having a second module in which they would have the chance to put the strategies and abilities they had learned into practice with their classmates:

*“The only thing I'd add to the course is...the opportunity to design a lesson and to try it out with our classmates, to see if we're taking the right path or not in the sense of developing thinking skills. I'd like to have taught a lesson. That's the only thing I'd add to the course, I liked everything else”* (Molly, Interview, December 2012).

I agree with them in this regard, because the teaching experience would have been more potent had the course been composed of two modules. During the first module, the prospective teachers would be the students and their learning processes would be achieved through the modelling of teaching practices, while the second module would allow them to take weekly turns to assume the teacher's role, which would give them an opportunity to design an activity with an explicit focus on the development of thinking skills, as well as to implement the activity they designed with their classmates. In this way, they would have had the time and the opportunity to develop and consolidate the reasoning required before building on this new understanding in order to increase their pedagogic content knowledge in this area.

In relation to teaching mathematics, most of the prospective teachers commented (9/11,  $f=31$ ) that, after the CAME course, they had a better view of what constitutes the teaching of mathematics and ways of promoting mathematics skills in their students in a more effective way than the manner in which they had been taught. The following illustrates this topic:

*The course made me stop thinking of teaching as dictating content. This bothered me a lot because, in the end, they [students] learn for the test and then forget the information. It's clear that you forget content that you're not going to use, because it doesn't make any sense to remember things that you don't use. So the course, in addition to teaching me mathematics, taught me how to teach it [mathematics]. I liked the process of teaching, maybe because it didn't teach me a particular mathematical concept...For me, it was more like, you can teach this content in this way. That's why it taught me how to teach (Amelie, Learning Journal, September 2012).*

The development of strategies and tools to teach mathematics could be related to the fact that most of the prospective teachers also felt more confident regarding teaching maths after completing the course (7/11,  $f=20$ ). While it is clear that this may have been partly due to their overall pedagogic development as



part of their teacher training course, their responses in the interviews indicated that they valued the CAME course as contributing towards this growing confidence in their teaching prowess. In this regard, Emily stated:

*“I do think that I now feel more confident about developing these skills in my students...because ...I had always previously tried to apply formulas in a very rigid way...in this course, I realised that you're allowed to use other abilities, to take other routes and to use different strategies to get to a result”* (Learning Journal, November 2012).

In this context, it is not surprising to find that teachers' confidence regarding teaching mathematics skills to their students improved, if they also think that the course equipped them with more strategies with which to teach mathematics.

According to the prospective teachers, the final contribution that the CAME course made in terms of their professional experience was to provide them with a repertoire of strategies that could help them in the future to motivate their students to learn mathematics (9/11,  $f=22$ ). At the beginning of the course, most of the prospective teachers claimed that they had had negative previous experiences with learning mathematics and, because of that, their motivation levels were very low when they realised that the CAME course was related to mathematics. However, during the implementation of the course, they encountered a new way of approaching and teaching mathematics, which actually allowed them to enjoy it more than they had expected. For this reason, if they liked the CAME methodology as students, it is understandable that they would transfer the CAME approach to their future classroom practice.

### **The case of Sarah: an example**

As I described at the beginning of this chapter, the presentation of results during it has the purpose of gaining higher levels of understanding with each

section. For that reason, I decided to include an illustrative case for two main reasons. Firstly, in contrast to the disaggregation and fragmentation that result from the coding of the data, exemplifying cases provide a coherent and unified image of the case being presented. Therefore, it assists in making sense of the data as a whole. In the second place, a case makes it easier to illustrate and exemplify the results presented in the previous section and to get the flavour of them, especially for someone who is not familiar with the original data. With regard to the advantages of including cases in educational research, Nisbet and Watt (1984), amongst others, argued that they communicate and provide understanding of essential aspects of the data that are usually lost in the whole and that, as they are very close to reality and to the core of the problem, they are useful for the understanding of and comparison with similar and/or different cases.

Before going into detail regarding the case of Sarah, I will present the reasons that I chose her from among the participants. At the beginning of the course, she was an average student in terms of age, reasoning level, attitudes towards mathematics and level of attendance. In other words, she was representative of most of the prospective teachers that participated in the CAME course. However, by the end of the course, she had made meaningful progress compared with that of her peers, both in terms of her reasoning level and of her attitudes and views regarding the teaching and learning of mathematics. Therefore, I felt it would be interesting to present the experience of an average prospective teacher who made better than average progress during the CAME course in more detail.

It is important to note that this illustrative case will be presented in the form of a story, because the purpose is not to show the way in which I reached the conclusions that I did, but to illustrate the views of one of the participants. Therefore, there are few quotations that complement my description. For more details regarding the conclusions I reached, the manner in which I conducted the analysis and further quotations related to the coding list, please refer to Chapters V (p. 144) and VI (p. 154). Having said that, I will now introduce Sarah's story.

Sarah is a 22-year-old student who is in the final year of her Bachelor of Education degree, which is the minimum certification requirement in Chile in order to qualify as a primary school teacher. She decided to participate in the

CAME course because it was presented by the Department of Education as a voluntary training opportunity to develop reasoning skills, which was an unfamiliar topic for her. When she attended the first session, she realised that the CAME course would be developed in the context of mathematics, which she believed was the most difficult subject in the curriculum; thus, she lost most of her initial motivation for attending the course. It is possible to say that Sara's negative reaction to the fact that the course was related to Mathematics was the most frequently observed among all the participants, since most of them ( $f=22$ ) had experienced negative learning experiences with the subject.

However, during the first CAME session Sarah, like most of her classmates ( $f=29$ ), was intrigued and attracted by the course methodology and decided to give it a chance. She eventually attended 10 of the 12 CAME sessions and, according to the Science Reasoning Tasks (Shayer, 1977), improved her reasoning level from mature concrete (2B) to early formal (2B/3A) thinking, likewise did 60% of prospective teachers who participated in the CAME course.

At the beginning of the course, Sarah told me that she had decided to become a primary teacher because she had had a very good experience at school, which was predominantly linked to having very good teachers. She describes her teachers as follows:

*"...very close to the students, very caring. They didn't go to the school just to teach, they were always worried about each of my classmates...if someone missed a school day, they'd call home and check if everything was ok"* (Interview, August 2012).

The exception to these good memories of her school teachers was her mathematics teacher, whom she remembers with annoyance and anxiety. Sarah described him as being authoritative and fixed in his views regarding teaching and learning. This teacher always wanted to impose his way of solving problems and did not allow students the opportunity to get to the same result by using a different method. In this manner, he maintained his 'expert' role in the classroom, and his approach to teaching was to demand that his students mimic the way in which he

worked mathematically. This gave the learners the impression that studying mathematics could only be achieved in a single, fixed way, with no flexibility in the approach. It was interesting to note that this feeling was shared by most of the participants ( $f=46$ ) when they claimed that maths teachers usually do not allow different processes to get to the same result or that they do not pay attention to the process only to the result. As a result, their experience as maths students was very mechanistic without placing emphasis on thinking, understanding, application or transference.

Regarding this point, Sarah remembers that as she did not easily understand mathematics, she looked for help at home but, during the tests, she usually got almost all the problems wrong, not because of the result, but because of the procedure, which was not exactly the same as that which the teacher had taught them. This is a very common trait in mathematics teachers. As she argues:

*“...for example, even though there are many different ways to divide, many mathematics teachers teach only the one they like. As a result, some students who do not completely understand that way of dividing will memorise the steps and will probably solve divisions correctly, but that doesn't necessarily mean that they have learned how to divide...what's worse, if they don't follow the exact procedure during a test, even though the result could be correct, the teacher will say that it's wrong”* (Learning Journal, September 2012).

She claims that she would like to repeat the positive experiences she had at school with her students in the future, and that this would include mathematics. It is interesting to note that what most participants want ( $f=26$ ) and do not want ( $f=32$ ) to become as a prospective teachers is primarily influenced by their experiences as students. As an illustration, throughout the following paragraphs, it can be seen that the aspects of teaching that Sarah criticises and praises are generally related to what she has seen and experienced as being useful for her own learning processes at school or at university, and not to what she has learned theoretically about the way that children learn.

Sarah and many of her classmates ( $f=18$ ) feel that the enterprise of being a good teacher is not at all easy, especially when teachers are faced with more than forty students, which is the case in many Chilean classrooms. Sarah is quite disappointed by what she has seen in her short professional experience and internships: unmotivated teachers that make a minimum effort and do not really care about their students reaching their full potential. She is convinced that the first requirement for promoting students' learning is to ensure that they are happy because, if they have problems at home, for example, and if teachers do not take these problems into consideration while teaching, those students would not be able to learn, even if the teachers were to deliver lessons at the highest level. The fact that emotions play a crucial role on students' learning process was a frequent belief observed in one third of the participants ( $f=12$ ), which might be related to the fact that for them having bad experiences related to mathematics predisposed them negatively towards the course.

For that reason, from Sarah's point of view, teachers must always create a positive classroom climate and care about each student individually in order to facilitate their students' learning. This is the image that she has of her own teachers, who were always there for her when she needed them. In this respect, Sarah claims that, after making sure that all the students are emotionally prepared to learn, teachers need to set the objectives of the lesson. According to Sarah, the current problem is that most teachers set lessons that are mainly content-related and which do not have skill or competency objectives. When the lesson is centred on the transmission of content, teachers tend to promote rote learning instead of meaningful learning. This topic is related to the fact mentioned by all participants ( $f=42$ ) when claiming the importance of teachers focusing on the development of different kind of skills and not only in the transmission of certain contents which is what they have seen in their own experience as students.

Even though Sarah's statements might lead to the conclusion that her views regarding teaching and learning are consistent with a more constructivist position, at the beginning of the course is still possible to observe certain contradictions. For example, she claims that memorisation is an initial and necessary step in every learning process:

*“As in every subject area, it is necessary to know mathematics to teach mathematics, [which means that you] need to memorise and repeat the content at the beginning and then begin to comprehend it. Once you comprehend it, you can analyse, experiment and use it in different situations and contexts”* (Learning Journal, August 2012).

This kind of contradictions was also observed in one fifth of the participants ( $f=12$ ). However, in the case of Sarah it was possible to observe a shift in her view because, at the end of the CAME course, she claimed that when teachers do not focus on students’ comprehension but only memorisation, the students will not really learn meaningfully. For example, they will not really learn division, because they will not understand the purpose of division, when to use it, and how to use it in other situations. This is why Sarah argues that teachers should not see their roles as merely giving students the information they need in order to learn. If this were all that teachers would need to do to promote students’ learning, students could learn such information by themselves, because all the information is now available on the internet; therefore, teachers would not be needed. On the contrary, instead of just transmitting information or content, Sarah claims that

*“...students need teachers that present them with adequate materials that make them think, which challenge and inspire them...”* (Learning Journal, December 2012).

It is significant that, after participating in the CAME course, Sarah describes what teachers should do in their classrooms by using the words *thinking*, *challenge* and *inspiration*. Even though I did not share the theory or the pillars behind every cognitive acceleration session with prospective teachers, they ( $f=41$ ) were able to perceive an important part of its essence, which could have been based on their experience as learners during the course. This could also be related to the previous suggestion regarding the ways in which prospective teachers’ learning experiences play a crucial role in shaping their views and beliefs

concerning the characteristics of effective teaching practices. This point will be developed in the next section.

With regard to the process of learning mathematics, it seems likely that Sarah like most of her classmates ( $f=28$ ) also experienced a change in terms of her attitude after the CAME course. During the initial interview I had with her, most of what she shared regarding her mathematics learning experience was tinged with painful and unpleasant memories. However, by the end of the term, she was able to mention some positive aspects of mathematics. More specifically, she stated that one good thing about mathematics is that, to some extent, it facilitates teachers teaching abilities and not merely content, because skills and competencies are a major component of mathematics. Another particularity of mathematics, she claimed, is that students need to learn it from the beginning because each component of content is necessary in order to learn the next. By contrast, one can finish a thematic unit and then start a new one that is not related to the previous one in other subjects, and this will not affect the students' future learning. Thus, Sarah believes that mathematics is much more hierarchical than are other subjects and topics that build on each other from year to year. With hindsight, she hypothesised that the reason that she hated mathematics was because she did not have a sufficient grasp of mathematical topics when she was younger.

With regard to Sarah's ideas that learning mathematics requires students to have developed previous contents and abilities, she claimed that, in her experience, when teachers realise that some of their students are not following a mathematics lesson, they usually continue with the class and ignore those who do not understand. As a result, those students will probably never understand mathematics. However, what teachers should do is to go back and check the misunderstood processes, because such mistakes are related to basic conceptual misconceptions, such as the concepts of number or quantity. The problem, she stated, is that the easiest thing to do is not to do anything about it. In this context, she recalled a mathematics teacher that she had in high school who, when Sarah said she that did not understand something, always replied that she had already explained the concept and kept going forward. From Sarah's point of view, these

are the kinds of experiences that students remember about mathematics and which cause them to hate the subject.

Thus, Sarah believes that she would attempt to remedy this situation by paying attention to her students' mistakes, as these could provide information regarding the processes that they have misunderstood. She argued that most teachers only look at the results and put a tick or a cross beside them, without realising that the process is much more important than is the result itself, because it gives better information regarding mathematical understanding. According to her, she learned this during the CAME course, because there was always room for different approaches to the same problem, as well as for going back to analyse and compare the different processes. This was one of the most valued aspects about the CAME course ( $f=51$ ). Most of the participants claimed that CAME methodology and the problems we addressed during the course always emphasised a flexible approach to it in the sense of giving the students the chance to solve it in many different ways or from different points of views.

In relation to this, Sarah claimed that

*"...the CAME course taught me how to learn mathematics in a different way...and that there are different methods of getting to the same result. It made me believe again in mathematics, that you could be creative when learning it. The way I used to approach mathematics was that there was only one way of doing everything and that's it, but the CAME course gave me the opportunity to analyse some problems individually and then collectively in order to compare our different reasoning processes. That was completely new for me"* (Interview, December 2012).

Another shift that could be seen in terms of Sarah's views regarding the process of teaching and learning mathematics is the recognition that is crucial for students to think about their own learning processes; if they do not do so, they are memorising rather than reasoning. As I illustrated previously using a quotation from Sarah's words, at the beginning of the course, she claimed that memorisation and repetition were necessary steps in the process of learning mathematics.



Nevertheless, after the CAME course, she claimed that, even though memorisation is an ability, it is a very basic one and, if students do not learn to think about mathematics, they will not be able to learn how to add, subtract, multiply and divide, nor to understand the functions thereof.

The CAME course contributed to prospective teachers' professional learning in that, upon conclusion, they were able to acknowledge the importance of developing students' thinking skills ( $f=42$ ). In fact, Sarah argued that if teachers do not strengthen their own thinking skills, they will not be able to promote those skills in their students and they will never be able to think mathematically. In this context, Sarah claimed that the CAME course gave her the skills and desire to teach mathematics differently in the future. She is convinced that presenting a challenging problem to her students and not giving them hints or instructions regarding the solution, as was the case during the CAME course, would improve their ability to reflect and to analyse. She also felt strongly that such an approach would facilitate in-depth learning, which is currently uncommon in mathematics classrooms. It is interesting to note that most participants ( $f=54$ ) mentioned the word challenge for describing the kind of reasoning and problems they engaged during the CAME course, because as I said before I never told them that it was one of the purposes of the approach and still they experienced it as learners.

According to Sarah, one reason that teachers do not usually encourage thinking in their classrooms is that because they are generally closed-minded and use pre-determined methods that do not allow students to get to the result using different processes. Even though she was not sure if the course helped her to strengthen her mathematical skills, she claimed that it changed her view of mathematics, because she realised that there was not only one way of solving each problem, as she had been told at school, but many different ways of approaching the topic. In this aspect, Sarah was different than the rest of the participants ( $f=64$ ), because even though she did improve her reasoning ability she was not aware of it. In contrast, most of the rest of the participants claimed at the end of the CAME course that they improved their ability to think, even those who actually did not ( $f=25$ ). This difference might be related to the fact that Sarah was specially insecure about her own capabilities from the beginning of the course,

and apparently the CAME experienced did not help her to increase her confidence levels.

In relation to the impact of the CAME approach for her future carrer as a mathematics teacher, Sarah mentioned that the course allowed her to believe that creativity could be part of the process of learning mathematics. As previously described, before the course, she thought that there was only one way of learning mathematics and that mathematics was merely a set of rules to be memorised and followed. She then came to believe that

*“if all mathematics courses were like this [CAME] course, everyone would like mathematics”*. (Learning Journal, November 2012)

Sarah also appreciated that the methodology we used during the CAME course promoted all working pairs or groups to approach and analyse the problem from different points of view and to use distinct strategies. In other words, what most participants commented ( $f=48$ ) was that the CAME course emphasised the process over result, which is in contrast to their previous experience of learning mathematics in which there was no place for analysis, only for mechanisation. According to Sarah, the CAME course helped her to understand that students learn a different way of thinking through mathematics, which is not only applicable to mathematics, but also to everyday problems. She described the CAME methodology as being very open, in the sense that it accepts different points of view and promotes the analysis and comparison thereof.

The two previous paragraphs are relevant for two different, but related, reasons. Firstly, they demonstrate Sarah’s lack of awareness of her own progress during the course in terms of the development of her mathematical thinking skills. In fact, she explicitly acknowledged whether the course had an impact on her mathematical thinking skills, even when she made a meaningful improvement on them in terms of the Science Reasoning Tasks, She moved from a Mature Concrete (2B) to an Early Formal (2AB/3A) reasoning level. Furthermore, she said that the CAME course taught her a different way of thinking that is not only applicable to mathematics, but also to everyday problems. This unawareness

might be related to the fact that Sarah always found CAME activities challenging. Thus, her view regarding developing mathematical skills could be that improving them meant finding the activities easier at the end than at the beginning of the CAME course.

Secondly, her lack of awareness in terms of the improvement in her mathematical thinking skills contrasts with the clarity with which she described the change in her vision and attitude towards mathematics. In fact, throughout the description of her case, it was common to find statements that describe the positive way that she now feels about mathematics and the teaching thereof, as well as the way in which she began to believe in the possibility of using creativity when learning mathematics.

Like one third of the participants ( $f=37$ ), Sarah claimed that one of the advantages of working in pairs or groups is that it gave her the opportunity to get to know her classmates' thinking processes. She described that, when she was listening to other approaches or solutions to the same problem, she often experienced the feeling of "*Oh! That's right!*", because they were explaining something that she had not thought about before. This usually occurred, not only within the group in which she was working, but also when each group shared their solutions and findings with the rest of the class later in the CAME session. Therefore, she argued that she enjoyed CAME methodology and that she believes that sharing and discussion is a central aspect thereof.

These statements might lead one to infer that students' learning experiences could be more powerful than regular teaching and learning practices in transmitting the essence of a methodological approach like CAME, which could have implications not only for training teachers in CAME, but also in any other professional development context. It certainly suggests that the participants ( $f=46$ ) were aware that a more socio-constructivist approach to teaching and learning was more influential in the mathematics classroom. Linked to this, Sarah also believed that working in groups had the advantage of generating a deeper comprehension and analysis, because she had to understand and evaluate her classmates' ways of solving a problem before dismissing her own.

In this context, Sarah claimed that working in pairs or groups instead of individually had the advantage of

*“[making] me open my mind and consider other possibilities or points of view. In other words, when my classmates started to give me their arguments, that opened a window of opportunity for me, and the same happened when the different groups were sharing their procedures...that would open another window”*  
(Learning Journal, October 2012).

In contrast, Sarah thought that if the person who proposed another approach were her superior and not her peer, in this case the teacher, she would accept it without question. However, in the case of the CAME activities, at the beginning she had thought that her way of doing something was best and, when her group did not agree, she had to listen to their arguments before reconsidering and making her own decision.

Sarah was part of the participants ( $f=42$ ) who thought that the CAME methodology helped her to enhance her metacognitive ability by promoting her awareness not only of her own thinking processes, but also by enabling her to recognise and analyse her classmate's thinking processes. She claimed that the fact that every problem was to be solved in pairs or groups played a central role in developing this metacognitive awareness, since she always had to share what she was thinking with her group, as well as having to listen to what the rest of the group was thinking and to then make a decision regarding the best way to approach the problem.

Another change that could be seen in Sarah was her comprehension of thinking skills and their purpose. At the beginning of the course, most prospective teachers ( $f=29$ ) were not able to give a definition or explanation of thinking skills. In fact Sarah openly admitted that, even though she had heard the term many times before, it was a topic that had not been covered during her initial teacher-training programme. However, at the end of the course, she understood thinking skills as the ability to apply and transfer previous knowledge to new and everyday life situations. Based on the kind of activities that the CAME course involved, it is possible to understand why she described thinking skills as the application of previous knowledge to new and real life situations. In fact, she

described CAME activities as being familiar and meaningful, because they presented hypothetical contexts that could occur in the course of a normal day.

With regard to her definition and description of thinking skills, she also claimed that they evolve from more concrete to forms that are more abstract. This assertion is, to some extent, consistent with the progression of thinking skills developed during the CAME course. In this sense, even though each CAME lesson tried to promote one or more thinking skills, the arrangement of lessons was designed to proceed from simpler to more complex thinking skills. In this context, it is significant that Sarah was able to perceive this, and that she included this point in her definition and characterisation of thinking skills.

Sarah recalled that when she learned that the course aimed at promoting thinking skills, she felt afraid and anxious:

*“...it made me feel nervous, because I thought that, as undergraduate students, we should have already developed those skills”* (Learning Journal, November 2012).

However, during the course, she realised that even though everyone talked about the importance of developing thinking skills, nobody had previously taught her these skills. According to her, the CAME course was her first introduction to a methodology that could help her to understand how to develop these skills in her students.

She also admitted that, prior to the CAME course, she had thought that thinking could be promoted in almost every subject except mathematics, and that she now realised that mathematics is an excellent learning context in which to promote thinking skills. This shift is related to her change in belief regarding mathematics. As described previously, after the CAME course she began to understand that CAME teaches students a different way of thinking that is not only applicable to mathematics, but also to everyday problems. This conclusion could be extremely powerful in shaping the way in which she might teach, particularly with regard to mathematics, during her future career as a primary school teacher.

This change is also related to Sarah's shift in confidence in terms of the teaching of thinking skills. Before the course, she did not feel confident about her ability to teach thinking skills, particularly to older students. Sarah thought that, in order to promote thinking skills, students needed to be motivated and, as many are not inherently motivated, teachers needed to motivate them, which was challenging to her. She also said that, when teaching thinking skills to older students, the abilities she would be teaching would be more complex. Therefore, teaching them would also be more complex, because such skills would be at a similar level to those that she possessed. However, the CAME course enabled her to feel more prepared to teach these kinds of skills and more motivated to try and to practice them until she felt ready to teach them.

With regard to her general ability to teach, Sarah stated that she felt nervous of the tremendous responsibility of being in charge not only of the learning process of an entire class, but also of their attitudes towards learning. Sarah's poor school experience of maths led her to believe that she would need to put extra effort into planning each lesson, as she could not stand in front of the class feeling insecure.

These statements not only reinforce the idea that what teachers do in their classrooms might have a strong influence on their students' attitudes towards learning in general and towards learning certain subjects in particular, but could also have an impact on prospective teachers' confidence regarding their ability to teach. This may be because prospective teachers have been students for a large proportion of their lives; thus, they could be particularly aware of the impact that teachers have on attitudes towards learning, creating tremendous feelings of responsibility that can be frightening, as is the case with Sarah.

In summary, it is possible that many of the assumptions and ideas that were present at the beginning of this thesis and which actually shaped it were confirmed by the case of Sarah. She not only changed her views about teaching and learning in general, but also about mathematics in particular, as well as developing a more positive and adaptive attitude in this regard. In addition, the CAME course helped her to advance her knowledge in terms of understanding thinking skills and ways of promoting them in her future career as a primary school teacher. Finally, CAME lessons contributed not only to the promotion of

higher levels of Sarah's reasoning skills, but also to her confidence regarding teaching in general, and of teaching thinking skills in particular. The aggregation of these results might suggest that using a cognitive acceleration approach in an initial teacher-training course is a powerful learning and professional experience for prospective teachers.

## VII. Discussion of findings and contribution to knowledge

### Implications for initial teacher training: the case of primary mathematics

#### *A. Prospective teachers previous views about teaching and learning*

One of the major findings of this research project was that the prospective Chilean teachers' views regarding teaching and learning mathematics are highly influenced by their experiences as students/learners. However, engaging the prospective teachers in thinking skill activities over several months during their training course changed their views about teaching and learning in general and with respect to teaching mathematics in particular. These findings are consistent with previous literature, which revealed that many teachers tend to teach in the same way in which they were taught as students, either at school or at university (Ball, 1988, Borko and Mayfield, 1995, Hill, 2000, Bruce, 2004, Henderson and Rodrigues, 2008).

According to Remillard (2000) is that teachers' previous ideas and beliefs were not taken into account,. The design and implementation of such initiatives has not considered that the core ideas they are trying to install are meaningless to many teachers, because they are forced to put into practice teaching methods that they have not seen in practice as students. Similarly, Schifter and Fosnot (1993) claimed that many teachers see changes in educational programmes as interesting teaching models, but do not know how to apply them to their own teaching experience because they are very different from what they are used to in terms of mathematics instruction.

For this reason, Brown et al. (1999) stated that, to some extent, teachers need to unlearn what they bring to their mathematics education courses in order to be able to break the vicious cycle of reproducing their experiences as students and to develop new teaching practices that are consistent with current educational reforms. Bruce ( 2004) takes this claim one step further by arguing that not only are prospective teachers' methodologies mainly the result of their previous school



experience, but so are their mathematical ideas and understanding. Here, Bruce (2004) claims that the concepts of mathematics, the teaching of mathematics and mathematics learning are closely interlinked.

This change in the view regarding the teaching and learning of mathematics was evident in the illustrative case of Sarah (please refer to Chapter VI, p. 196), when she described what she felt that teachers should or should not do in their classrooms in terms of teaching approaches and in which she elucidated the best way of promoting students' learning. At the start of the study, her views were heavily influenced by her experience at school as a student but, through her involvement with the CAME intervention, she was provided with a wider range of learning experiences upon which to draw and reflect. Thus, she was able to conceptualise both learning and the role of the teacher in supporting learning in a manner that differed from her previous views.

Prior to the intervention, Sarah, like other prospective teachers, thought that mathematics was merely a set of rules and procedures to be memorised and applied accordingly, based on what she had experienced in terms of teaching and learning mathematics during her school and university experience. However, after the intervention, it could be seen that she had begun to understand and conceptualise mathematics from a different point of view, because she realised that mathematics, as with any other school subject, involved thinking if it were taught and presented to students in an appropriate way. Even though this evidence is interesting and has relevance in relation to the design and implementation of teacher development sessions, further exploration would be needed in order to gain confidence and to improve understanding of how one might incorporate a thinking skills approach into classroom practice.

These conclusions and the evidence from this and other studies presented in this research should alert teacher educators to the factors that might have an important role in expanding teachers' future practice. While this does not mean that such practices are not changeable, it does imply that initial teacher training instances are compelled to take this evidence into account at the time of designing and implementing courses in order to provide prospective teachers with adequate experience to develop the abilities, capabilities and knowledge required to promote their students' learning potential in the classroom.

### *B. Prospective teachers' previous mathematics learning experiences*

Finding that teachers tend to teach in a similar way to which they were taught is a problematic conclusion, particularly when considered in conjunction with the fact that most prospective teachers who participated in this study had negative prior experiences of mathematics, either at school or at university. In many of these cases, the unpleasant memories were associated with the teachers they had had during their trajectories as mathematics students and/or learners. The number of times that prospective teachers recalled negative experiences with mathematics in contrast to the number of times they mentioned good memories was startling.

Mathematics anxiety and a profound dislike of maths is a common and well-documented phenomenon among prospective primary teachers (Quinn, 1997, Tooke and Lindstrom, 1998, Zettle and Raines, 2000, Vinson, 2001, Sloan et al., 2002, Uusimaki and Nason, 2004, Uusimaki and Kidman, 2004, Bursal and Paznokas, 2006, Gresham, 2007, Gresham, 2008). According to Vinson (2001), mathematics anxiety is characterised by

- i. Uneasiness when asked to perform mathematically (for example, to divide a restaurant bill)
- ii. Avoidance of maths classes until the last possible moment
- iii. Feelings of physical illness, faintness, dread or panic
- iv. Inability to perform in a test
- v. Utilisation of tutoring sessions that provide very little success.

Many scholars, including (Levine, 1996, Martinez and Martinez, 1996, Raymond, 1997, Hart, 2002, Uusimaki and Nason, 2004) have associated mathematics anxiety with prospective teachers' previous learning and teaching experiences. The problem is that teachers who hold negative attitudes and beliefs regarding mathematics might produce students who have similar feelings about the subject (Sovchik, 1996, Zettle and Raines, 2000, Vinson, 2001, Sloan et al., 2002, Uusimaki and Nason, 2004, Gresham, 2007). In addition, negative attitudes

towards mathematics have been related to poor performance in mathematics (Post, 1992, Gresham, 2007), which might create a vicious circle of negative attitudes that could result in low achievement, which would reinforce such previous, negative attitudes.

Some educators (Tobias and Weissbrod, 1980, Furner and Berman, 2004, Uusimaki and Nason, 2004, Gresham, 2007) have related the development of mathematics anxiety in students to traditional teaching techniques that are frequently used in mathematics classrooms, such as organising the class structure according to students sitting individually in rows in front of the blackboard, not differentiating student work by difficulty level or performance, not promoting understanding but merely memorising, seeing mathematics as a set of rules, devoting most of the lesson to the whole-class format in which the teacher talks and the students listen without participating and emphasising that there is only one correct approach to solving a problem, amongst other factors.

It is interesting to note that one of the most frequent criticisms that participating teachers made of conventional mathematics instruction was the fact that teachers tended to overemphasise the result at the expense of the process. In this sense, they reported that teachers did not usually allow different ways of getting to the same answer, and that mathematics is usually taught in a mechanical/theoretical way that does not value thinking, understanding, application or transference. Participating teachers also thought that this aspect of their previous mathematics instruction experience was related to their negative concept of and attitude towards mathematics. This description is consistent with the traditional practices of teaching mathematics that previous research has associated with the development of negative attitudes in this regard (Tobias and Weissbrod, 1980, Furner and Berman, 2004, Uusimaki and Nason, 2004, Gresham, 2007).

By contrast, a number of non-traditional or infrequent teaching and learning methods have been documented as not only playing a crucial role in diminishing students' negative attitudes and beliefs regarding mathematics, but also as promoting positive feelings and beliefs towards it, such as problem-based learning (Gresham, 2007), collaborative group work (Beswick, 2006), relating mathematical experiences to the student's real-world environment (Gresham,

2008), reflection (Hart, 2002), simulations, discoveries and challenges (Seymour, 1996, Vinson, 2001, Gresham, 2007), among others.

In this context, the socio-cultural experience of the CAME intervention for the participants of this intervention group allowed them to experience and to conceptualise the process of teaching and learning of mathematics quite differently, because what CAME tries to do is to present challenging problems to the students that they have to solve in a collaborative way with their classmates. In this sense, the exchange of different points of view and discussion among students is key to accomplishing the objectives of every CAME lesson. In relation to group work, Blatchford et al. (2003) claimed that, in the future classroom, students working together will be key to the process of learning from and with each other, because we live in a society in which enormous amounts of information are broadly and instantaneously available; thus, the most appropriate learning scenario is one in which groups of students make sense of this information together. For this reason, teachers need to promote their students' collaborative practice in order for them to understand, make sense of and make use of that information in a meaningful way. This requires a more socio-constructivist approach in the classroom than that which the prospective teachers would have encountered in either their school or their university experiences. Blatchford's (2003) research team also pointed out that another benefit of learners working collaboratively is that their achievement improves considerably, in comparison to when they are working on an individual basis.

Based on the characteristics identified by previous research, it can be hypothesised that the approach to teaching and learning encapsulated in the CAME course could be related to the observed change in views regarding teaching and learning in the prospective primary teachers. The CAME methodology complies with many of the reported features, in that it presents challenging, real-life problems to students that they have to solve in small groups and about which they must reflect in terms of their progress, as well as ways in which the processes could be improved and transferred to other situations or contexts.

As mentioned previously and described in more detail in the results section (see Chapter VI, p. 154), it was only at the end of the CAME course that the

prospective teachers began to criticise the way in which mathematics is usually taught. Taking part in the intervention changed the ways in which the prospective teachers viewed and valued the teaching approach to mathematics. In this sense, after the CAME course was complete, they claimed that traditional mathematics classrooms emphasised the results to the detriment of the processes and mistakes; therefore, learning and understanding the reasons behind mathematics received little attention. Furthermore, teachers do not usually allow the development of different processes in order to achieve to the same result, but frequently demand that students follow exactly the same algorithm as was demonstrated during the class. The prospective teachers also indicated that maths is usually taught in a mechanical/theoretical way that does not promote thinking, understanding, abilities, application and/or transference.

All the claims that prospective teachers made regarding the CAME course support the idea that it changed their beliefs regarding effective mathematics teaching and learning by experiencing this for themselves. Based on these findings, it is possible to suggest that their participation in the CAME course gave them the opportunity to contrast what they had previously been told about teaching mathematics, what they had seen of teaching and learning mathematics in practice and what they experienced during the CAME course. As a result, they reached the conclusion that teaching and learning mathematics could be much more meaningful and interesting than they had previously experienced during their trajectories as mathematics learners at school or at university.

In fact, even though the prospective teachers may have been introduced to constructivist approaches to mathematics teaching during their studies, most of the participants explicitly reported that, at the end of the CAME course, the methodology was very different from that which they have previously experienced with regard to teaching and learning mathematics in practice. Specifically, they claimed that the methodology used during the course was new to them and was very different from what they were used to in other mathematics courses at university or at school. The recognition that CAME methodology was not only innovative but was also attractive for prospective teachers might have influenced the development of attitudes and views towards teaching and learning mathematics that were more constructivist and more positive. This could be

considered to be a potent consequence of the course.

According to Hart (2002), beliefs are the basis of teachers' practice. Consequently, in order to change and affect teachers' practice, it is first necessary to deal with their beliefs. With reference to this point, Pajares (1992) stated that "the beliefs teachers hold influence their perceptions and judgments, which, in turn, affect their behavior in the classroom, or that understanding the belief structures of teachers and teacher candidates is essential to improving their professional preparation and teaching practices". The problem is that most of the teachers' beliefs are already formed when they enter teacher preparation programmes, because beliefs are usually developed according to previous experience.

Therefore, Hart (2002) proposed that, in order to have a real impact on teachers' practices and to develop the necessary constructivist views that new educational trends demand, teacher preparation courses should involve prospective teachers in teaching and learning experiences that put these principles into practice. Hart (2002) also claimed that one of the most frequent mistakes that initial teacher education makes is teaching mathematics content not only separately, but also by using traditional teaching methods like lectures that are not consistent with the constructivist principles that are usually taught during the mathematics methods courses. As a result, prospective teachers are not able to inform their future practice based on their own experiences as learners; thus, the likelihood of changing their beliefs is dramatically decreased.

The findings from previous research make perfect sense in the light of the results of this study, which point out that one of the aspects of the CAME course valued by prospective teachers and which was explicitly reported at the end of the course was the fact that they learned by actively participating in CAME lessons, by handling real life mathematical problems and by following a methodology that was consistent with a constructivist view of teaching and learning. These claims are not surprising when taking into consideration the fact that all cognitive acceleration programmes are theoretically and practically driven by constructivist principles. However, as this was never made explicit to the participants in this course, it is interesting to note that not only did they recognise this aspect, they also commented on its value.

All the evidence presented and discussed so far has significant implications for the development of effective initial training programmes for prospective teachers, since it suggests that the methodologies used during teacher preparation courses have as a strong an impact on teachers' future practice as do the actual content and teaching/learning theories covered in such courses. With regard to this point, Adler et al. (2005) stated that most teachers of mathematics have not yet developed the necessary skills and knowledge that the teaching practice will demand of them and, more importantly, have not yet learned them in ways that would be useful for teaching their future students efficiently. For this reason, it is imperative not to talk about the principles behind the new educational trends during teacher preparation courses, but to incorporate them into the methods used during these courses in order to give prospective teachers the opportunity to experience, as learners, that which they are supposed to impart as teachers.

### **Implications for promoting thinking skills in initial teacher preparation courses**

The results of this study suggest that the three initial teacher education programmes that participated in this research might not sufficiently promote the development of reasoning skills. These results are consistent with previous research in the field (Leat, 1995, Lee, 2005, Barak and Dori, 2009, McDonald, 2010). According to Barak and Shakhman (2008), teachers need not only to have good knowledge of subject matter, but also need in-depth pedagogical knowledge in order to promote student's thinking skills as general skills, in addition to the specific skills in the subject that they are teaching. This research showed that not only was the methodology developed in this intervention novel and unfamiliar to the students from the three participating universities, but also that the topic of thinking skills was new to them and that they presented a lack of professional knowledge regarding thinking skills and how they could be promoted in their classrooms.

When prospective teachers were asked to provide a definition of thinking

skills, most of them were unable to do so. It was therefore evident that this was the first time they were dealing with the topic of what defines thinking skills and ways in which they could be promoted in the classroom. It was also clear that the knowledge to which they were referring when attempting to provide a definition of thinking skills was based on informal and general knowledge, rather than on knowledge developed in a professional context as part of their training course as prospective teachers.

It is interesting to note that, even when prospective teachers did not have a clear idea of thinking skills and their promotion in school students, that they claimed that teachers are not currently or sufficiently emphasising these skills in their classrooms. These claims might be related to the fact that they were aware that CAME methodology is aimed at encouraging the promotion of thinking skills, and that the methodology was somewhat different from that which they had encountered during their trajectories as students. As a result, they not only claimed that this learning goal was not currently being accomplished in traditional classrooms, they also recognised the complexity involved in this objective.

As described in the literature review section (see Chapter III, p. 45), the lack of reasoning skills observed in prospective teachers has led to some scholars stressing the relevance of promoting thinking abilities during initial teacher education programmes (Cox, 2007, McDiarmid et al., 1989, Kennedy, 1990, Reynolds, 1992), since they are not only essential for conducting good quality teaching practice, but also for improving these skills in their pupils. In relation to this, Peterson and Treagust (1995) stated that teacher education programmes should concentrate on the development of prospective teachers' pedagogical reasoning ability instead of overstressing the development of content knowledge. Only through this change of focus will teachers be equipped with the necessary understanding of the content, the curriculum and the learners that they require in order to make meaningful and effective decisions regarding their own teaching practice.

In this context, it could be argued that the impact of the CAME course on prospective teachers was relevant not only because they had improved their performance in the reasoning skills test by the end thereof, but also because they reported that participating in the course helped them to realise that they were able



to solve mathematical problems, which they began to enjoy, or improved their understanding of mathematics. In addition, they claimed that they had a better idea of the process of learning and teaching mathematics and of how to promote maths skills in their students in a better way than that in which they had been taught. They also reported that the course helped them to become more aware of their own learning and thinking processes. With regard to the CAME lessons, they said that they had fun during them, that the activities were motivating and that they could use these types of activities to motivate their students in the future. Finally, they commented that the course had been useful in terms of providing them with the necessary tools to design and orientate their lessons according to the promotion of different thinking skills, and that they had used their own reasoning skills to solve the problems.

Based on the impact and the positive reception that the CAME course had on participating prospective teachers' thinking skills, it could be suggested that taking the cognitive acceleration approach is a viable strategy for improving initial teacher training courses in Chile in the future. In this regard, Fennema et al. (1996) agreed that promoting teachers' knowledge and understanding of their students' thinking is a feasible approach for changing and improving teachers' mathematical instruction, because they could use this kind of information to inform and to change their previous teaching practice.

Even though all those findings are significant, it is important to note that it was not possible to observe the prospective teachers working with students in the classroom; thus, while they stated that their confidence regarding teaching and promoting thinking skills with their students had substantially improved, I cannot be sure that this is actually the case. Nor can I be certain that their views regarding the importance of teaching thinking skills for long lasting, transferable, applied or meaningful learning were sufficiently stable and capable of influencing their future practice in the classroom. As stated in the previous section, one of the most effective ways of changing teachers' practices, beliefs or attitudes, is through experiencing those changes and principles in practice (Hart, 2002).

This might be one of the reasons that prospective teachers did change meaningfully from their perspective as students, but not necessarily as meaningfully as future teachers. Each CAME lesson lasted between 50-60

minutes, which is actually the normal duration of any cognitive acceleration lesson and the time allowed was sufficient to fulfil the learning objectives set for each specific lesson. In other words, there was enough time to introduce the activity, to present the challenge, to give students time to solve it, to discuss what they did, to reflect on how they did it and try to find new contexts in which those strategies could be applied. However, there was not enough time for prospective teachers to try out the new tools they had developed during the course. This was actually one of the few criticisms they had of the course. As a result, the experience of participating in the course as learners might have been a more potent and meaningful learning experience than the tools that they might have developed for their future careers as teachers.

This does not necessarily mean that the course will not have an impact on their future practice as teachers because, as described in the previous section, they developed attitudes towards mathematics that were much more positive and they also moved towards more socio-constructivist views regarding teaching and learning, which suggests that this will affect their teaching practices in a positive way. This supposition is consistent with the conclusions reached by the cognitive acceleration team based on the follow-up they made of the professional development (PD) programme they offered with every cognitive acceleration intervention programme. According to Adey (2006), if teachers really want to change their practice in order to encourage their students' thinking skills, they cannot follow rigid procedures or a set of rules. Instead, cognitive acceleration methodology requires teachers' awareness of their beliefs about teaching and learning in order to be able to question them and, if necessary, to change and adapt them to more coherent approaches with a cognitive constructivist approach to teaching and learning.

Comparable results were found by a research project called Cognitively Guided Instruction (CGI), which attempted to explore whether teachers' beliefs regarding teaching and learning mathematics would change after participating in an initial teacher training course based on students' thinking. According to Vacc and Bright (1999, p. 90) Cognitively Guided Instruction (CGI) is

...an approach to helping teachers use knowledge from cognitive

science to make their own instructional decisions. Children's knowledge and the teacher's understanding of that knowledge are central to instructional decision making. Teachers plan instruction using research-based knowledge about children's mathematical thinking and well-defined taxonomies of problem types and children's solution strategies for arithmetic operations. Teachers seek specific information about individual students' thinking and understanding and then adjust the level of content to match individual students' performance levels.

Similar to this study, the CGI project found that, after participating in the pre-service course, prospective teachers moved from views that were more aligned to the transmission of information format to one that was more consistent with constructivist principles regarding the teaching and learning of mathematics (Vacc and Bright, 1999).

Even though the results presented so far are significant and are consistent with the results of other similar studies, there may have been a greater impact on the confidence levels of prospective teachers in terms of promoting thinking skills in their future students if the course had included a structured and specific time for practicing the tools and strategies of the CAME methodology within their own teaching practice. In fact, one of the prospective teachers' views about thinking that had changed by the end of the course was their awareness of the complexity of fulfilling this teaching objective. In other words, they realised that promoting thinking skills in their classrooms is a challenging learning goal, which might be the first step towards trying to find other ways of acquiring the confidence they need to fulfil that goal. According to Barak and Shakhman, "the real challenge of in-service training courses for teachers is not in the teaching of new instructional methods, but rather in increasing teachers' self-confidence in their ability to introduce these methods into the class wisely and continuously" (2008, p. 202).

## **Implications for cognitive acceleration research**

### *C. The need for more research on teacher change within the CA context*

This research study has shown that participating in a CA training instance is a powerful and meaningful learning experience for pre-service teachers. However, the specific focus of this study is relatively new because, even though some research has been conducted based on the evidence of the professional development programme that each cognitive acceleration intervention runs (Adey, 1995, Hodgen, 2003, Adey, 2004b, Hodgen et al., 2004, Johnson et al., 2004, Adey, 2006, Hodgen and Askew, 2007), the quantity of such research is considerably less than is the number of studies that have focused on the effects of CA intervention on school students' thinking skills and academic achievement (Adey, 2005, Adey et al., 2002, Adey and Shayer, 2002, Adey and Shayer, 1990, Cattle and Howie, 2007, Endler and Bond, 2008, Mbano, 2003, Adhami et al., 1997, Iqbal and Shayer, 2000, Shayer and Adhami, 2003, Shayer and Adhami, 2007, Shayer and Adhami, 2006, Shayer and Ginsburg, 2009, Shayer et al., 2007, Shayer et al., 1999, etc.).

In addition, those studies that have tried to explore the effects of CA professional development programmes on teachers have mainly been motivated by explaining the effect in relation to students' cognitive gains and improved performance. In other words, there have been relatively few studies (Hodgen, 2003, Hodgen et al., 2004, Johnson et al., 2004, Hodgen and Askew, 2007) whose main objective has been to specifically analyse teacher change and to explain why, how and what the most important or salient impacts of the PD programme on them are.

For this reason, it would be interesting to develop future research initiatives that would investigate teachers' processes of change within the context of CA training programmes in order to increase the understanding of the underlying processes of such change. This increased understanding made it easier to enhance the experiences of the teachers who participate in such programmes.

#### *D. The new horizons of Cognitive Acceleration programmes*

While I was designing the CAME course and adapting the activities, I was not sure if that adaptation was going to be successful in the sense of the activities being adequate, motivating and sufficiently challenging for working with prospective primary teachers in Chile. This uncertainty was partly because this was the first time that the CAME materials were being used with prospective teachers, but also because the lessons were to be delivered in a completely different cultural context. Even though the adapted materials were well received and the early implementation during the pilot provided significant feedback regarding ways of improving the final version of the materials, the uncertainty remained during the entire implementation process.

Another inherent complexity of the intervention was the double objective of generating significant learning experiences for prospective teachers in their roles as students and as future teachers. At the beginning, it was not evident that designing a thinking skills course for prospective primary teachers would generate meaningful experiences. A further consideration was the choice of the mathematics version of the Cognitive Acceleration programme, which implied further challenges for the success of the implementation of positive attitudes towards mathematics among the prospective teachers.

Despite all these reservations, the group of findings presented in the Results section (for more details, see Chapter VI, p. 154) provided relevant evidence that supports my original belief that a CAME course could provide prospective teachers with powerful learning experiences, both as mathematics students and as future teachers. In fact, prospective teachers who participated in the CAME course not only developed attitudes towards mathematics that were more positive and views about teaching and learning that were more constructivist, but also valued the structure of the lessons by making explicit references to the main principles behind them (cognitive conflict, social construction and metacognition), even though I had not explained the theoretical

bases of CA activities to them. Adey (1999) refers to these principles as the pillars of CASE.

The first main pillar of CASE is cognitive conflict, which consists of the confrontation of students' current assumptions and ways of thinking through the presentation of a problem (for more details, see Chapter III, p.53). With regard to this pillar, the participating prospective teachers mentioned that the CAME course presented challenging activities or problems that forced them to use their thinking skills in order to solve them and, even when the proposed problems were placed in the context of mathematics, they realised that it was not sufficient to make use of regular algorithms, formulas or procedures, but that they required processes that were more complex in order to get to a solution.

The second main pillar is social construction, which claims that children's cognitive development or mental growth is essentially a social process and, for this reason, collaborative group work is one of the most important characteristics of any CA lesson because it gives students the opportunity to create and to share knowledge with their peers (for more detail, see Chapter III, p. 53). This CA feature was also clear to the CAME course participants, because they claimed that sharing their reasoning and processes was useful in terms of being able to learn from their peers and from their own mistakes, as well as to become conscious that there are different kinds of thinking and learning styles. They also claimed that the methodology helped them to verbalise their own thinking patterns.

The last pillar is metacognition, which implies that the cognitive acceleration approach intentionally promotes participating students to think and to reflect on both the individual's and the group's thinking processes. This aspect originates in the idea that children should come to think of themselves as 'thinkers' who have some control over their own thinking processes. The ability and the disposition to reflect on how one has solved, or even failed to solve a problem, is a powerful tool that enables children to have more control of their own learning (Adey, 2008). Many of the prospective teachers commented that the course helped them to become more aware of their own learning and/or thinking processes.

Even though some aspects of the course were more significant and produced a greater impact than others, based on all the findings presented so far, it

is impossible to deny that using the CA approach for developing a prospective teachers' course was a successful training experience with many positive impacts. Cognitive Acceleration programmes have proved to be a significant approach with students of different ages in a variety of schools subjects and in different countries and cultural contexts. For this reason, my claim is that it is worth finding different and novel contexts, other than that of school students, in which this kind of initiative could be applied and which might contribute to the development of thinking skills that are so desperately needed in this globalised information era.

### **Contribution to the existing body of knowledge**

#### *E. Exploring the results of CAME with populations other than school students*

Given the particular characteristics and demands of modern society, an increasing number of educational initiatives are oriented towards the development of children's cognitive abilities (De Bono, 1991, Heathcote and Bolton, 1994, McGuinness, 2000, Claxton, 2002, Trickey and Topping 2004). However, one of the positive aspects of the Cognitive Acceleration approach is not only that it is an established intervention that has been implemented successfully for more than 30 years (Shayer and Adey, 1981), but also that its impact on students' achievement has been measured rigorously and systematically. In this context, there is a large body of research (Adey, 2004a, Adey et al., 2002, Adey and Shayer, 2002, Adey and Shayer, 1990, Adhami et al., 1997, Shayer and Adhami, 2003, Shayer and Adhami, 2007, Shayer and Adhami, 2006, Shayer et al., 1999) that reports the cognitive and academic gains that students have made after participating in Cognitive Acceleration programmes.

In the specific case of the Cognitive Acceleration in Mathematics Education programme (CAME), the research team found that, even though the results obtained by the experimental students immediately after the completion of the project were not very big in terms of effect sizes, they were relevant compared

to the achievement of the comparison group, both in terms of the mathematics test and the General Certificate of Secondary Education (GCSE) for maths, science and English. In fact, the results showed that, for students who attained the higher levels, the gains were significant because three of the experimental classes had twice the number of students performing at the grade C level or above. In this sense, with regard to the gains reflected on the mathematics post-test that was administered, they predicted greater added value for GCSE grades three years later (Shayer and Adhami, 2007).

The results of this study are consistent with previous evidence, which shows that participating in a CA programme is a meaningful and powerful learning experience that enables students to move towards higher reasoning levels. However, this specific research had the particularity of delivering a CAME programme to a completely new population, namely prospective primary teachers, in a completely different country- Chile. This is one of the contributions of this thesis, because it shows that taking a CA approach could be a successful decision for improving and promoting the development of reasoning skills, not only in school students but also in older populations, and even in other cultural contexts in which the spoken language is not English.

I have to admit that the observed changes that prospective teachers made in terms of their reasoning skills were far beyond my expectations. As described earlier in this thesis (for more detail, see Chapter III, p. 53), regular CA programmes have generally been delivered across two school years and, even within that time frame, many of the effects have been observed years after the completion of the intervention (Shayer and Adhami, 2007). For this reason, from the beginning, I was extremely cautious and conservative regarding the claims I made in this regard and this was reflected in the research design. As I was not expecting to observe significant changes in prospective teachers' reasoning skills as measured by the SRT test, I considered the quantitative aspect of this project being secondary. As a result, I focused most of my attention on the understanding of the participants' underlying processes of change during the CAME course. This could be considered a limitation of the study and will be further addressed in that section (for more detail, see Chapter VIII, p. 231).



Even though the group of prospective Chilean teachers who participated in the CAME course made statistically significant gains in terms of their reasoning skills in comparison to those of their control counterparts, I remained cautious regarding the scope and the generalisation of these results. It is important to take into consideration that CAME lessons were originally designed to be used with KS3, S2 and S3 school students in the UK. As a result, it may seem unlikely that undergraduate students could benefit from them. This is a legitimate question and, from my point of view, even though the methods used in my study were adequate and rigorous, the results cannot be completely disconnected from the context in which they were obtained.

It is widely recognised among educational leaders that prospective Chilean teachers are not being sufficiently prepared to promote their pupils' learning (Avalos, 2003, Contreras et al., 2008, Ortúzar et al., 2009, Bellei and Valenzuela, 2010, Montecinos et al., 2010, Peirano, 2010, Cabezas and Claro, 2011). Based on this evidence, it might be possible that the observed gains were related to the low level of reasoning skills that prospective teachers had at the beginning of the intervention. In other words, using the same CAME course with older populations who are better prepared may not produce the same statistically significant results.

#### *F. Using a mixed methods approach within the context of CA research*

Taking a mixed methods approach is already a contribution to the research in the educational field. As described in the Methodology chapter (for more detail, see Chapter V, p. 96), social reality is highly complex and this complexity may be better understood by approaching it via different methods (Husén, 1988). In spite of this, some researchers, such as (i.e. Gorard, 2002), have argued that the mixed methods approach is the least frequent method used in the context of educational research. In this situation, having conducted a mixed methods research project in which quantitative and qualitative results are consistent, it is not only beneficial to the educational research field from a methodological point of view, but also from

an educational standpoint because the results might be considered to be more robust than if they had been explored from a single perspective.

In addition, using both quantitative and qualitative methods within a CA research context is a novel approach. Traditional CA programmes have generally preferred quantitative methods, such as quasi-experimental designs, the measurement of effect sizes and so on, in order to explore the impact of the intervention on participating students. By using these kinds of methods, CA researchers have been able to document the magnitude of the gains that experimental groups have made after CA programmes, both in terms of cognitive skills and of academic achievement, in comparison to those of their control counterparts. Nevertheless, this approach is not particularly informative in terms of explaining how such gains were achieved, as well as other processes that are behind or related to them.

In fact, after Shayer and Adhami (2007) developed a CAME project in 12 schools in the UK, they found significant results not only in terms of their reasoning levels, but also in terms of their academic achievements in science, mathematics and English, the research team hypothesised that the set of CAME activities might not in itself be the only explanation for the students' progress. In this context, they claimed that what CAME might be doing is encouraging and promoting the professional development of the teachers who participated in the intervention. It was explicitly suggested by the trainers that the teachers needed to make connections between the cognitive aims of the CAME activities and their teaching during 'normal' mathematics lessons.

Consequently, teachers began to make use of the same teaching skills across the curriculum and students faced an approach that was more focused on thinking during their learning process. This may have produced a virtuous circle that eventually had a positive impact on students' cognitive development and learning (Shayer and Adhami, 2007). However, these claims were not further explored because the research was focused on measuring school students' change in terms of their reasoning skill levels and their academic achievement. In this context, I consider that developing a CA research project that took a mixed method approach might have favoured the understanding of other psychological

and social processes associated with the development of these higher reasoning levels.

In this sense, as described in the Results chapter (for more detail, see Chapter VI, p. 154), having the chance to discuss prospective teachers' views regarding teaching and learning in general and about mathematics in particular before and after the CAME course gave me the opportunity to witness the way in which their views moved towards a socio-constructivist approach to teaching and learning, as well as how they gained confidence about their abilities to teach. These findings, in conjunction with the fact that prospective teachers improved their reasoning levels, make a relevant contribution to the understanding of the underlying processes of prospective teacher change and cognitive development.

## VIII. Limitations, learning and future research

### Limitations of this research project

This study not only provided evidence that shows that it is possible to deliver a Cognitive Acceleration programme to populations that are older than school students and to observe improvements in terms of their reasoning level, but also to have a deeper understanding of other psychological and social processes involved in these changes. However, it would be naïve to think that all the changes observed in prospective teachers during and after the CAME course might be completely explained by the experiences they had during the sessions. Therefore, this section will present some of the limitations of the CAME course in terms of explaining the improvements observed in the experimental group.

#### *A. The limits of CAME's explanatory power: time and other training instances*

In order to analyse the scope of CAME in accounting for the improvements observed in the participants, it is important to bear in mind that, when an intervention is trying to promote a certain kind of cognitive development as is the case of CAME, that development is necessarily influenced by the natural maturation process that occurs in every person over time. In other words, it is almost impossible to establish a dividing line between the changes that are explained by the direct or indirect effect of the CAME intervention, and the magnitude of the change that is attributable to the time that has passed.

In this context, the time (five months) that elapsed between the beginning and the end of the CAME course is one of the variables that could interfere with the explanatory potential of CAME course. However, every intervention study has this limitation and, therefore, this research used a control group in order to be rigorous and reliable in terms of exploring the effect of time. In this sense, not finding the same improvements in the control group in comparison with the

experimental one (for more detail, please refer to Chapter VI, p. 154) might be considered a sufficiently strong argument in favour of the impact of CAME.

In addition to the time, the fact that prospective teachers were participating in other university courses that were part of their Bachelor of Education programmes is another limitation for the explanatory potential of CAME. As described previously (see Chapter V, p. 127), the CAME course was offered as one of the optional courses that prospective primary teachers could choose from among the selection of courses that their Education Departments were offering during that semester. In this sense, prospective teachers were engaged in other courses related to mathematics and to teaching primary children as part of their training to qualify as primary teachers.

As a result, it is impossible to be sure that the changes observed in the experimental group were the sole result of my intervention. Again, in these kinds of cases, the main tool that provides insight into the effectiveness of the intervention is having a control group that allows the comparison of the experimental participants with similar students (control participants) under similar conditions. Therefore, the fact that the control group did not improve their reasoning skills to a similar extent as did the experimental group over the same period of time is still an argument that backs up the effect of CAME.

### *B. Limitations associated with the administration of the Science Reasoning Task*

As described in the methodology chapter (see Chapter V, p. 133), one of the measures that were used to explore change in terms of reasoning level was the Science Reasoning Task test. The results showed that the experimental group made statistically significant improvements in comparison with the control group. However, it was strange to find that the control group did not change at all in terms of their SRT test performance in the pre- and post-tests, which is very unusual (for more detail, please refer to Chapter VI, p. 154). Even though I returned to the original tests and the scoring process to double check the results, I realised that they were correct.

After reflecting on this unusual issue, I could think of two possible explanations. Firstly, the control group, unlike the experimental group, had no reason to be motivated to put effort into answering the Science Reasoning Task (SRT) questions. In other words, the experimental group saw the SRT test as part of the “duties” they needed to fulfil for participating in the CAME course, in contrast to the comparison group that generously agreed to participate in a research project by taking a test without expecting anything in exchange. In fact, I witnessed this situation when I was administering the test to the control group, and I realised that most of the prospective teachers devoted little time to answering each question with the purpose of leaving the room as soon as possible. This phenomenon might partly explain the lower results obtained by the control group in the SRT.

Secondly, the SRT test has typically been used to measure the effects that CA programmes have had on students’ reasoning skills after two years. In other words, in traditional CA programmes, experimental and comparison groups have taken the test at least twice: at the beginning and at the end of the intervention two years later. In this context, it is possible to hypothesise that the SRT test might not be sufficiently sensitive to register small changes in terms of reasoning skills, such as those experienced by the prospective teachers from the comparison group five months after taking the pre-test. In fact, five of the fifteen control participants obtained the same reasoning level in the pre- and the post-test, even when all of them experienced some change in terms of the number of correct responses in each of the tests.

Another explanation for the progress the experimental group made in comparison with the control one is that the experimental participants became more competent at answering the test and that their reasoning levels had not necessarily improved. However, I am not fully convinced of this hypothesis, because both groups took the test twice and, as I have explained elsewhere, the SRT test has been validated and widely used for this purpose within the context of CA research. Furthermore, the data were analysed from different points of view in order to ensure that the statistical differences were real and were not just the result of the kind of test used.

### *C. Limitations related to the sample size*

Moving to other limitations of the research project, it is possible to say that the small sample size was problematic, particularly for drawing statistical inferences. At the beginning of the research process, I was going to recruit a substantial number of experimental and control participants in order to be able to compare them and to make inferences regarding their differences. However, when I started the recruitment process, I realised that it was going to be a much more difficult task to recruit participants than I had anticipated. In the case of the experimental participants, being part of the research project involved coming to a CAME session every week for the entire university semester and also involved taking a test at the beginning and at the end thereof (for more detail, see Chapter V, p. 127). Even though all the participants were free to leave the course at any time during the semester, the conditions of participation implied a high-level and long-term commitment right from the start. This might be one of the main reasons that could explain the difficulty in recruiting a larger sample.

The control group was also difficult to recruit, especially because their participation was limited to taking a test at the beginning and at the end of the semester. While the experimental group experienced certain benefits as a result of their association with the CAME course, the control group did not perceive any benefits in taking the test so, even when a higher number of students agreed to take the test at the beginning of the semester, it was extremely difficult to persuade them to return a second time at the end of the course. As a result, I had no other choice than to consider the experimental and control participants from all three universities as only two groups, without being able to make comparisons between universities but only across them, even though they were, in fact, quite dissimilar.

#### *D. The consequence of conducting research within a PhD timeframe*

As previously mentioned in the Results section, another limitation of the study was the limited timeframe (five months). CA programmes in any subject usually last for two school years, because the aim is to accelerate the development of cognitive structures that are more complex and cognitive development takes time. Even though the prospective teachers made statistically significant progress in terms of their reasoning skills, it is important to remember that this course was not a traditional CAME course, but was a course that was specially adapted for prospective teachers. In this sense, it had the simultaneous objective of improving their thinking skills and equipping them with various tools that could help them in their future practice as primary teachers.

The limited time frame meant that the participants did not have sufficient time to transfer the new skills they experienced as university students to their teaching practice, and nor was I able to research this aspect. In other words, the experience they had as university students during the CAME sessions might have been more meaningful than their experience as future professional teachers. I only became aware of this issue at the end of the course when I had the opportunity to talk to the students about their learning experiences. Therefore, even though the initial idea behind the intervention was to equip teachers with the tools they needed in order to be able to use this system in their future classrooms, my methodology mainly allowed me to explore their experience as students and I was not able to follow up on their professional development as future primary teachers.

Evidently, this limitation is not only related to the research design itself, but also to the inherent limitations of conducting research within a PhD time frame. If I were to comply with the allocation of time for the PhD, it was not feasible to run an intervention that would last two years, even though I was aware that teacher change is slow and requires concerted effort (Adey, 2006). Nevertheless, this does not mean that I did not believe that running a CAME course that lasted for one semester would not be a significant learning experience for prospective teachers. Had I not believed this, I would not have implemented



the course. On the contrary, I was so convinced of the potential benefits of the CAME course for prospective teachers and of the limited time available, that I took into account this information when designing this research, when choosing the methods and for deciding the kind of research questions that could be answered under such conditions.

### **What I learned from conducting this research**

This was the first time that I was in charge of a research project in which I took all the decisions. From the beginning, I was surprised by the fact that, even when I planned every aspect of the research process carefully, many of them were different from what I had initially expected. This does not necessarily mean that everything was more difficult than I had planned, but merely that it was different in the sense that some things that I thought that would be very complicated to achieve were actually easier, while others that I thought would be easy were, in fact, quite difficult to accomplish.

Based on this experience, by the end of the research project it was clear to me that every research process is completely dynamic and that many aspects thereof are unpredictable. Therefore, as a researcher I have to be tolerant of the aspects that did not turn out exactly as I expected but, most importantly, the researcher must always be extremely analytical in order to reflect on the outcomes during, and not after, the research process, and to make the necessary decisions that will allow him/her to answer the research questions as satisfactorily as possible. In other words, I learned in practice that the planning and design of a study does not end before the study begins, but that any research project is an iterative process and that the act of planning, designing, evaluating and re-designing has to be present throughout the entire process.

Previous paragraphs described the conclusions that are applicable to the research process as a whole. However, there are specific aspects of this research that I would have done differently if I had the chance to start the process all over again. Firstly, some of the complexities of my recruitment process were associated with the fact that it was not possible for me to reach prospective

teachers directly, as I had to first obtain institutional approval from the Departments of Education (DEd). Only after receiving the DEd's agreement to participate in the research project was I able to get into contact with the potential participants and invite them to participate in the CAME course. However, none of the three DEds gave me initial direct access to their students' contact details. Therefore, I had to write an explanatory e-mail that contained the details of the course I was offering and details of their anticipated participation, which the DEds then forwarded to prospective teachers.

This meant that, when prospective teachers made a decision regarding whether to participate in the CAME course, they did not have all the necessary information and nor could they ask questions about the course, because the DEd had the same information as the prospective teachers. In fact, many of the prospective teachers that decided to participate in the CAME course admitted that they had no idea what the course was going to be like before they started attending the sessions. From my point of view, this lack of information might have hindered the recruitment process, because the kind of information I would have been able to give them via e-mail was not as clear as it would have been had I been able to talk to them and to answer their questions.

In this sense, at the beginning I did not expect that the DEd were going to act as intermediaries between the students and myself. However, as this turned out to be the case, I will anticipate this in future and would consider having us sign an agreement in order to establish certain ground rules. For example, an issue that arose during the process was related to the fact that I managed certain confidential information regarding the participants, such as the level of attendance at the course and the reasoning levels measured by the SRT tests. Even though I stated in the participants' information sheet that I would not share this kind of information with other university authorities or teachers, I did not make this clear to the DEd in a formal way (by means of a written agreement), but only told them that I was not willing to disclose such information to them. I did not have any problems related to this issue in the end, but I still think that it would have been better if I had seen the Education Departments as research participants as well and thus followed the same formal guidelines and procedures, such as an information sheet, consent form, that I followed with the prospective teachers.

Managing my relationship with the DEd in a better and more formal way could also have had the benefit of letting them know what I expected of them if they agreed to participate in the research project, as well as what they could expect from me. For example, I thought they were going to provide me with photocopies of the note sheets and the activities for each lesson, but not all of them did that. In addition, I told them that I needed a room with movable chairs that would allow me to organise the students into groups of different sizes depending on the activities that developed in each lesson, but not all the DEds were able to comply with this request. It is important to note that I am not trying to suggest that DEd did not do this because they were unwilling to do so, but because they were unable to do so. However, it would have been better if I had known this from the beginning in order to be better prepared to make the necessary amendments when working with the prospective teachers.

Another thing that I would have done differently is to place greater emphasis on the development of higher reasoning skills measured by the Science Reasoning Task test. In the beginning, I claimed that measuring prospective teacher change in terms of their reasoning skills was a secondary focus of this research, because the focus was on understanding the underlying processes related to such change. However, the decision to set aside the aim of measuring prospective teachers' reasoning skill levels at the beginning and at the end of the CAME course was mainly justified by my belief that I was not going to observe considerable changes in this regard, since the allocation of time was not sufficient for promoting significant cognitive development. It is important to clarify that, as I said before, even though I did not expect significant improvements in terms of the reasoning skills of the prospective teachers that participated in the course, I was convinced that their participation would be a meaningful learning experience for them, even if this were not reflected in their SRT scores.

Finally, if I had the opportunity to redesign the course in a research context different from that of a PhD thesis, I would plan two different modules. The first would be oriented towards participants experiencing the CAME activities as learners, while the second would give prospective teachers the chance to design and to try out some activities as teachers with their classmates as students. This would possibly allow the participants to model the type of learning

they experienced during the CAME course within their teaching practice and to better prepare them to introduce these pedagogic ideas into their teaching.

As described during the results and discussion chapters, that first aim was more successfully accomplished than was the second. I am not suggesting that this is unrelated to the lack of time I experienced during the course, but I do think that the second objective of promoting teaching skills for the development of students' thinking skills in the future was not as strongly emphasised as the first during the CAME course. Clearly, such an endeavour is larger in scale than a PhD study could accommodate, but possibly working more closely with the DEd could result in the course being taught while only certain aspects thereof are accessed for data collection.

### **Some suggestions for future research in the field**

Cognitive Acceleration programmes have been implemented since the 1980s and have subsequently produced promising results in terms of students' cognitive gains and academic achievements (Adey and Shayer, 1990, Demetriou et al., 1991). For this reason, even when the first CA programme was delivered within the context of science, CA interventions began to be developed in other school subjects (Adhami et al., 1998, Shayer and Adhami, 2003, Adhami et al., 2005) and with students of different ages (Adey et al., 2001a, Adey et al., 2002, Shayer and Adhami, 2003, Adhami et al., 2005), and in various countries (Iqbal and Shayer, 2000, Mbano, 2003, Endler and Bond, 2008).

As a result, CA became not only a successful intervention for promoting students' thinking skills, but also a highly structured approach to teaching and learning that was frequently associated with rigorous follow-up and research processes. From my point of view, this is one of the salient characteristics of CA research because, within the educational field, there are many teaching/learning models that are successful inside the classroom but that do not transcend those limits because nobody knows about them. In contrast, the case of CA is very informative because most of the new initiatives have been closely linked to research and, as a result, their impacts have been rigorously documented.

In this context, it is somewhat difficult to state that trying out a CA approach adds completely new evidence to the existent body of knowledge, or that there are still many unexplored lines of research, because CA research has been active for more than 30 years. However, I am modestly convinced that this research may add some useful insights to the corpus of CA research, and that there might be some interesting approaches that could still be developed.

As described in a previous section, every CA programme has two lines of action. The first involves the application of the CA activities instead of regular lessons once a fortnight, with the purpose of promoting students' thinking skills. The second, which is equally important, is the implementation of a professional development (PD) programme for teachers in order to enhance their understanding regarding the theory and the principles behind the CA approach and to promote the skills they need in order to implement the programme with their students.

In comparison with the huge amount of CA research that has tried to measure the impact that CA acceleration programmes have had on students' thinking skills and academic performance in other subject areas (i.e. Adey, 2005, Cattle and Howie, 2007, Endler and Bond, 2008, Mbano, 2003, Adhami et al., 1997, Shayer and Adhami, 2007, Shayer et al., 1999, etc.), little research has been primarily interested in exploring the impact that the PD programme has had on the teachers participating in it (Hodgen, 2003, Adey, 2004b, Johnson et al., 2004).

Based on the evidence that CA research has produced and the findings of this thesis, it is reasonable to think that in-service teachers might have experienced significant changes in terms of their views and approaches to teaching and learning after participating in the PD programme, as the prospective teachers did. In other words, the success of the PD programme could rely not only on in-service teachers becoming capable of delivering the CA programme effectively, but also on them moving towards more productive approaches to teaching and learning. However, this line of research is relatively undeveloped, because regular CA studies have mainly tried to explore the effects of the PD programme in relation to students' cognitive gains and improvement of performance.

Only a small group of studies (Hodgen, 2003, Hodgen et al., 2004, Johnson et al., 2004, Hodgen and Askew, 2007) have implemented research projects aimed at understanding CA teacher change and to determine which other psychological and social processes play a relevant role in explaining how such change was accomplished. In this context, developing other studies that attempt an in-depth exploration of the process of teacher change within the context of CA professional development programmes could make a relevant contribution not only to the development of a better understanding, but also to the improvement of such initiatives.

Even though I said I was convinced that offering prospective teachers the opportunity to participate in a university course that used the CA approach might be a significant learning experience for them, I was not sure if the adaptation of the activities would be adequate in terms of creating the necessary challenges and motivation inside the classroom. These doubts were mainly related to the fact that using the CA approach with prospective teachers meant that the population targeted was going to be much older than were the previous, traditional CA students. In other words, I assumed that, to some extent, prospective primary teachers in Chile had not yet developed the formal reasoning skills described by Piaget and Inhelder (1958).

In addition, I was also aware of the implications of delivering a CA programme that lasted for only one semester, as the standard duration of these interventions is two school years. In this context, my expectation as a researcher was not to observe relevant improvements in terms of the participants' thinking skills, but to prove that prospective teachers valued the experience of participating in the programme, because it gave them the opportunity to approach the teaching and learning of mathematics from a completely different point of view. Traditional classrooms usually deal with mathematics in a very rigid and mechanical way, which contradicts the dialogic and collaborative work that CAME classrooms try to encourage (Adhami et al., 1998).

However, as described in the results and discussion sections, the findings of this research exceeded my expectations. Prospective teachers not only changed some of their views regarding teaching and learning to more socio-constructivist ones, but also developed more positive feelings about teaching mathematics, as

well as showing statistically significant improvements in terms of their reasoning skills as measured by the SRT test. In this sense, if such important processes took place within a limited time frame, I am convinced that developing further programmes with populations other than school students might also have promising results. This would be particularly relevant if such programmes followed the traditional structure of CA interventions in terms of lasting for two years and having a greater and more structured quantitative aspect in tandem with the qualitative approach for impact.

The last aspect I think would be worth exploring is to keep developing these kind of initiatives in other cultural contexts, in order to find out if it is possible to claim that the CA approach is universally successful and is not dependent on the country in which it is applied. In addition, it would be interesting to replicate the experience with other groups of prospective teachers from other countries because, to the best of my knowledge, this research was the first experience with these kinds of students.

## **IX. Summary and concluding comments**

This study explored how prospective primary teachers perceive and respond to a thinking skills approach to teaching and learning. The context of the study was three initial training programmes (Bachelors of Education) in Chile. This study is important because the political and historical events in Chile have resulted in teacher training courses that do not prepare prospective teachers to focus on learning issues. In this context, I was convinced that offering a university course for prospective teachers that could increase their use of thinking strategies, as well as introducing them to a successful intervention in cognitive acceleration methodology, might be a significant and fruitful experience that could help them to focus on the way in which their pupils learn.

The experience of using a cognitive acceleration approach with prospective teachers was successful, because prospective teachers changed their views regarding teaching and learning in general, and about mathematics in particular. While many of the participants had previously held negative views regarding learning in mathematics and quite fixed views about the teaching approach in mathematics, after the CAME course, the prospective teachers' views changed dramatically, leading to an increase in their confidence in mathematics and in expanding the types of approaches that could be taken in the primary mathematics classroom. This was achieved through allowing the participants to reflect on their experience as learners during the course, as well as on ways in which they could apply the methodology as teachers in the future. For the majority of the participants, the CAME course developed their confidence about teaching mathematics and changed their opinions regarding the importance of developing thinking skills in their classrooms. This means that the CAME intervention affected the prospective teachers both as learners and in terms of their future teaching abilities.

In terms of the results, this study supports previous research that claims that the views and beliefs that prospective teachers hold regarding teaching and learning are mainly shaped by their past experiences as students, both at school and at university. It is very important to have this kind of information in mind when designing and developing teacher training, both at pre-service and in



professional development instances for teachers. It is useful to some extent for this generation of teachers to reflect on and to benefit from the practices of their own teachers, as this ensures the continuity of educational ideas. However, a problem arises when there are new and innovative approaches to teaching that require teachers to break away from specific practices. In order to change the cycle of teachers teaching in the same way they have been taught, they first need to deal with the background and past experiences that teachers bring with them to the training situation (Brown et al., 1999).

In addition, if the training is to succeed in developing new and more complex skills in teachers, teachers have to see the new ways of working in practice, realise the benefits thereof and have the opportunity to try out those tools for themselves (Remillard, 2000). While my study enabled prospective teachers to get a taste of how the cognitive acceleration materials worked in the classroom, there was insufficient time and opportunity for these teachers to try out the materials with their own students. Nevertheless, prospective teachers reported that engaging with these activities changed their views on teaching and learning, particularly with regard to mathematics.

This study showed that the relevant experiences that prospective teachers bring to the training situation are associated with negative and unpleasant feelings and memories about mathematics. These results are consistent with prior research in the field (Uusimaki and Kidman, 2004, Bursal and Paznokas, 2006, Gresham, 2008). The fact that these bad experiences have been linked to traditional teaching tools that are frequently used in regular mathematics classrooms (Furner and Berman, 2004, Gresham, 2007) might be the reason that prospective teachers found CAME methodology both novel and motivating. Firstly, the CAME approach promotes collaborative work between students with different levels of performance. Secondly, it encourages students to articulate and to discuss their ideas and to try to find a shared solution. Finally, it also tries to emphasise the process rather than the result which, coupled with the other features, make it a more sociable and enjoyable approach to learning mathematics.

Some of the characteristics of the CAME methodology have been linked by others (i.e. Beswick, 2006, Gresham, 2007) with the promotion of more adaptive and positive feelings about learning mathematics. In this regard, it is

possible to understand why, after participating in the CAME course, prospective primary teachers changed not only in terms of their views about teaching and learning in general and about mathematics in particular, but also began to strongly criticise how mathematics is typically taught in regular classrooms. This approach is more in keeping with a constructivist approach to learning than are the types of scenarios the prospective teachers described regarding their own experiences of learning mathematics at school. This might have led to the conclusion that, by the end of the course, the prospective teachers held different ideas regarding what characterises an effective mathematics teacher and how the classroom should be in order to promote meaningful and positive learning in the students. The claims prospective teachers made about the CAME approach in terms of its novelty and attractiveness might be related to the fact that they moved to more socio-constructivist and positive views about mathematics.

It is interesting to note that participating teachers not only found that the CAME approach was new and attractive, but also that dealing with the topic of thinking skills during a university course was highly unusual. As a result, they also commented that they were unsure if they possessed the necessary knowledge and teaching skills for promoting these kinds of abilities in their students in the future. However, at the end of the course, they began to criticise the fact that thinking skills are not currently being promoted in regular classrooms in Chile. Thus, being involved in the cognitive acceleration course changed their views about the role of thinking skills in the school curriculum.

This study also found that the impact of the CAME course not only promoted a better understanding of what constitutes thinking skills in prospective teachers and how they could be promoted inside the mathematics classroom, but also the improvement of prospective teachers' thinking skills as measured by the Science Reasoning Task test. This is a significant result. While SRTs have been used extensively to map cognitive gains following cognitive acceleration interventions, this was the first time that they were used with prospective teachers. In this context, the results of this study suggest that it is possible to use the Science Reasoning Task test as a measure of students' reasoning levels in different contexts, cultures and age groups, which is a novel and relevant finding for the field of educational research as a whole.

In addition, this study not only documented the first experience of using a cognitive acceleration approach with populations other than school students, but also used rigorous methods to show that there were statistically significant differences between those prospective teachers who participated in the CAME course and those who did not. These results are original and important because they suggest that the cognitive acceleration approach is a valuable strategy for promoting the development of thinking skills far beyond the context in which it has traditionally been used. For this reason, the cognitive acceleration approach should be used with confidence in new and different contexts to keep promoting the thinking skills that are so in demand in this challenging educational era.

**Cada acto de aprendizaje consciente requiere la voluntad  
de sufrir una lesión en la propia autoestima.  
Es por ello que los niños pequeños, antes de ser conscientes  
de su autoestima, aprenden más fácilmente.**

**Thomas Szasz**

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## Appendix

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## A. Pilot Study: Summary of CAME activities selected

N°	Title	General Description	Thinking strands	Curricular links
2	Tournaments	An activity based on systematic listing, repeated addition, and multiplication as a short cut, set in the context of school sports tournaments. In the second part of the lesson pupils explore the use of symbols and expressions.	Systematic enumeration of combinations Comparing representations	Exploring algebraic symbols and expressions
15	Circle Functions	Pupils explore the area of the circle in relation to the radius (or diameter) and then compare circumference and area as linear and quadratic functions of the radius.	Reflection on concrete experiences of linear and quadratic functions and on their graphical representation.	Area or perimeter of a square as a function of side; $\pi$ ; Continuity.
20	Heads or Tails	Pupils record the number of heads they get in five samples of 10 tosses of a coin. They contrast the variation in the percentage of heads in their samples of 10 with the way the percentage approaches more and more closely to 50% as the class data is cumulated. This introduces the idea of sampling variation.	Models of probability and chance	Data-handling: probability and proportions
22	Comparing correlations	The emphasis is moving on from the intuitive notion that a wide scatter means poor correlation to comparing correlations quantitatively. Three different data-sets are compared by reducing them to four-cell tables of confirming and disconfirming cases. Correlation is estimated as a ratio of confirming and disconfirming cases.	Correlation as a degree of fit between one variable and another	Data handling: media and range; estimation and precision.
30	How do I handle the data?	An activity of selecting modes of data representation appropriate to different contexts. All the contexts call for comparisons of data sets, but the meaning are best revealed in different ways: either a direct visual contrast, graphs of functions, or correlation scattergrams.	Metacognition and bridging	Handling data: correlation, functions, modes of representation. Organizing and analysing data.

(Adhami et al., 1998)

## B. Pilot Study: CAME activities selected

# 2 Tournaments

An activity based on systematic listing, repeated addition, and multiplication as a short cut, set in the context of school sports tournaments.

In the second part of the lesson pupils explore the use of symbols and expressions.

### Thinking strands

Systematic enumeration of combinations  
Comparing representations

### Curriculum links

Exploring algebraic symbols and expressions

### Resources

Notesheets 1 and 2  
One calculator per group

## Lesson summary

### 1 Introduction (5 min)

Set the problem up for the class. Teams from three schools are playing in a sports tournament, where each team plays every other team at home. How many matches are played in total?

Don't give any particular way of organising the count. Ask how pupils would show on the page that no game is missed or counted twice. Give out Notesheet 1, where pupils should use their methods with four and more schools.

### 2 Pupils' work (10 ± 2 min)

Circulate to help individuals and encourage them to help each other. Note instances where the method of 'doubling', which produces the right total for three schools is recognised as not valid for other numbers.

### 3 Class discussion (10 min)

Elicit the different methods of enumeration, and their pros and cons. Move then to generalisation through use of different and large numbers, then to any number. The generalisation should first be made in words, then in symbols.

### 4 Introduction to Notesheet 2 (5 min)

Start with two examples for the same expression. For example, an uncounted amount of money in a purse from which 2p is taken can be described by the expression  $(n - 2)$ . The same expression can stand for an uncounted number of pupils who remain seated when you ask two pupils to stand up.

### 5 Pupils' work (10 ± 5 min)

Circulate, encouraging pairs or group work in creating good stories for the expressions.

### 6 Class discussion (10 ± 5 min)

Elicit various stories for each expression.

Conclude by repeating the message about symbols and expressions serving as number variables. Use various phrases, for example: number holders, number carriers, standing in for any number, representing any number, envelopes in which any number can be hidden.

Question 8 on Notesheet 2 may be used as an extension or as optional extra work for interested pupils.

## Mathematical content

The context for the first activity, based on Notesheet 1, is schools playing sports matches against each other. If every team plays all the others at home, how many matches are played in total? This is a case where the items to be counted have to be listed or represented in a systematic way. Different ways of listing and representing have advantages for different uses. For the purpose of finding the total number of matches, pupils should recognise that some

operations have advantages over others (in this case multiplication is more efficient than repeated addition). For the general case of 'any number', symbolisation is gently suggested.

In the second activity, based on Notesheet 2, the use of symbols is explored with emphasis on the symbol as a generalised number and its use in expressions. This part of the lesson starts with a class discussion with some instruction.

## Pupils' thinking

The mathematical content provides another context within which pupils can work towards the language of generalised number. It also develops the pupils' concepts of multiplication. The generalisation is achieved through recognising that the different methods of listings and representing contain the same two variables: the number of schools involved, and the number of games each school plays at

home. To find the total number of games played the repeated addition can be replaced by a more general multiplicative relationship expressed in words or in algebra.

Part of the agenda for the first activity is to give an opportunity for as many groups as possible to have something worth saying, and which they can demonstrate and discuss.

## Specimen lesson

### 0.00 Introduction

The teacher starts by explaining that the lesson will be about schools playing games with other schools at home grounds, and how to find the total number of games played from the number of schools. What game should they use as example? The class chooses Basketball by a vote.

The teacher elicits names of three primary schools where some pupils came from and lists them on board in a line: Northfields, Trees, Oldham. She asks each pupil to show on a piece of paper their way of finding the number of games played when each of the three schools plays the others at home. She explains that it is important to organise the work, listing or diagrams so as not to miss any game, or count any game twice.

While pupils list or draw their solutions the teacher encourages comparisons between different methods. When most pupils have managed a solution she gives out Notesheet 1 and asks them to work now with four schools. She stresses that they should start with their own preferred method and switch to another only if they find it easier.

### 0.05 Pupils' work

Some pupils make guesses with the total '8 games' for 4 schools thinking that each school still plays 2 games as in the example with 3 schools. The teacher encourages them to check by listing or drawing. When they realise their mistake, she asks how they did that themselves, and praises their orderly way of counting or listing.

Some pupils use **abbreviations** for schools, some use **colours**, and others **matchstick figures**. The teacher compliments the originality and asks if these make the counting easier and what other purpose they are serving.

For question 2, with 5 schools, some pupils **group** the schools or draw lines around them. Some write the number for each group. These presentation methods indicate some recognition of the generality of the problem, in that it appears similar to other problems which the pupils have solved in the past.

Some pupils start by using lines or arrows **between** two columns listing the schools or **within a single list**.

A variety of methods of grouping and listing used in the class are shown below and on page 10.

For question 3, with 6 schools, some pupils successively add six 5s in some way to find the answer, recognising **addition** as a short cut to counting. Some also use the 'times' operation, recognising **multiplication** as a short cut to repeated addition.

In question 4, the step of going from a solution for a known number of schools to an expression for any number of schools is fraught with difficulties. The teacher allows pupils who reach the question to struggle with it for a few minutes before moving to discussion.

### 0.20 Class discussion

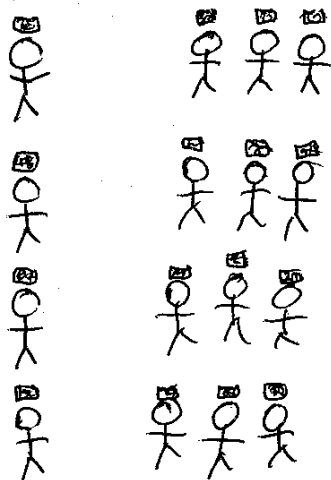
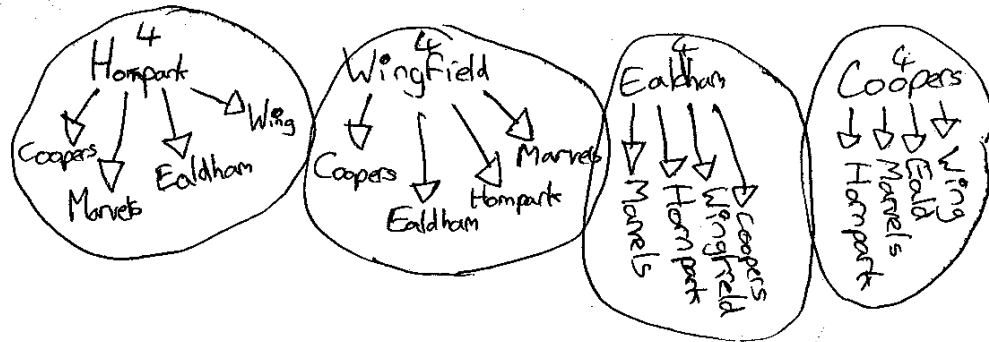
Class discussion focuses on the different methods used, rather than the answers. The teacher conducts the discussion without stressing a preference for multiplication. Each pupil should recognise that he or she has a method and that other methods can be used. The teacher stresses that pupils may prefer one method now, but some other method in a different problem, because it is more convenient.

The teacher lists the methods used by the pupils: the guess with a reason, the list, the grouping, the arrows and lines, the additions and the multiplication. She borrows some pupils' Notesheets to copy their diagrams on the board. Each method

### Samples of pupils' work for Question 2, Notesheet 1.

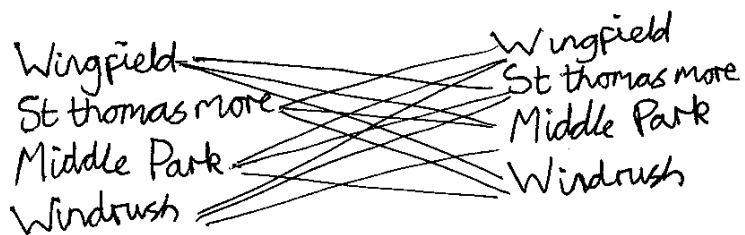
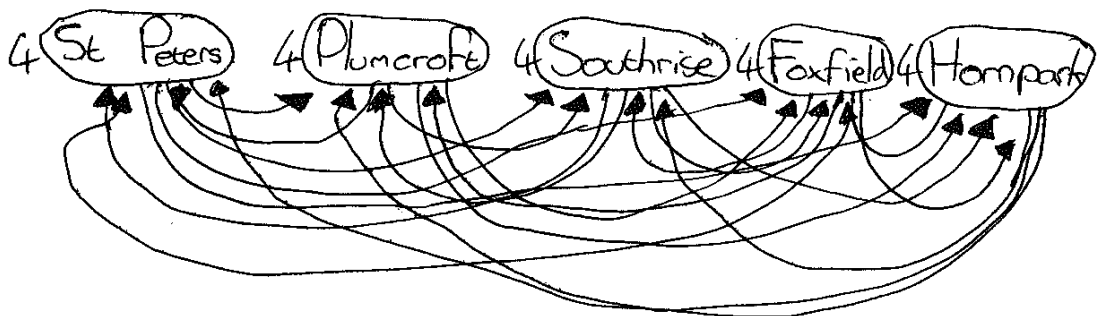
Coopers	Horupark	Marvels	Baring	Ealdham
Horupark	Coopers	Coopers	Coopers	Marvels
Marvels	Marvels	Wingfield 5	Wingfield 5	Horupark
Baring	Baring	Baring	Horupark	Wingfield 5
Ealdham	Ealdham	Ealdham	Ealdham	Coopers
Wingfield 5	Wingfield 5	Horupark 5	Marvels 5	Baring 5





$$3 \times 4 = 12$$

$$\text{or } 4 \times 3 = 12$$



$$\begin{array}{r} 55 \\ 55 \\ 55 \\ \hline 165 \end{array} \quad 30$$

A range of grouping methods pupils used in the TM2 Tournaments specimen lesson

is then looked at in terms of advantages and disadvantages.

The advantage of a **guess with reason** over just a guess is that the reason gives a starting point for checking and realising your own error.

The **list** is best to make sure everybody is counted. However if the number of schools is large, so much writing can lead to mistakes. The list must also be systematic and organised.

**Grouping** the schools can make listing and counting easier. Adding another school means adding another group, and also adding one to each group.

**Lines and arrows** make for less writing, and contain the same information as lists. They are mathematically more elegant but may be untidy. An easy way of counting games is to look only at the ends of the lines coming out of each school name (or coming to it), so you have groups of lines. As with the lists, if you add a new school you add one more line to each of the groups, and a new group.

**Using abbreviations** or symbols is a short cut for writing and makes listing groups easier.

The teacher emphasises: 'Number of Schools', and 'Number of Games each school plays' for each method, highlighting the way they are grouped. She then asks: "What is common between all the methods?" She extracts, with some prodding, that all the methods show the games each school plays and that the total number of games comes either by repeated adding (5 games for each of the 6 schools, 6 games for each of 7 schools etc.) or by 'times'-ing (6 lots of 5, 7 lots of 6 etc.) The teachers dwells on the equivalence of these two calculations, realising the difficulty of multiplication for many pupils at this age.

### 0.30 Teaching episode

A teaching episode follows on how to find the number of games for any number. Starting with 10 schools, moving on to 20 schools, then to 50 and 100 schools, the teacher gets from the class the number of games each school plays at home, then calculates the total number of games. Some pupils use calculators to check. Then the teacher uses the symbol  $n$  for the number of schools, and the expression  $n - 1$  for the number of games each school plays at home. The teacher reminds the class they have used variables and expressions like these before to stand for any number.

### 0.35 Introducing Notesheet 2

The teacher explains that symbols are used for many different things: abbreviations for full names, or

labelling flats or rooms A, B, C. She reminds the class that some pupils used letters as abbreviations for school names in their work in Notesheet 1. In all these cases the letter stands only for one thing. But in Algebra we use a letter to stand for **any number**, which is very different. And when you make an expression like  $n - 1$ , that also can be any number, depending on what number  $n$  takes.

She explains that  $n$  can be a number of any things: money, or people or schools or pencils. She then gives an example. She gets some coins from her purse in one hand and takes 2p out. She asks: "If I had 40p here before, how much do I have left? What if I had 80p? 77p? 13p? Any number of p's?" She gets them to accept  $n - 2$  for that situation.

She quickly gives another example for the same expression. She asks two pupils to stand up, and asks: "If there were 30 pupils sitting before May and Rad stood up, how many are still sitting? What if there were 24 pupils sitting before? 20 pupils? 130 pupils?" She gets the class to realise that the coin and the pupils examples are similar in some way. They can be stories for the expression  $n - 2$ .

### 0.45 Notesheet 2

Notesheet 2 is given out. The teacher asks for ideas for question 1, and then for question 2, before asking the class to write their own stories.

Circulating, the teacher encourages pupils to work in pairs and select the best stories they come up with for discussion.

Some pupils have great difficulty with the idea of generalised number and the teacher uses questions such as

"What can letter  $n$  stand for here? Give me a possible number. Another number?"

"What number can letter  $b$  stand for here? So what does 2 times  $b$  make?"

### 0.55 Class discussion

Pupils share some of the stories they have made for each expression. The teacher picks one from each group.

The teacher praises the class thinking. She suggests some pupils may want to try question 8 at home and come back to her with their stories. This question asks pupils to suggest symbols for a number of things themselves, make an expression by adding or subtracting some number from that, and then to find stories that fit the expression.

## **2** Tournaments

### Notesheet 1

- 1** 4 schools play in a tournament. Each school plays all the others at home.

List the four schools,

..... ,  
..... and .....

How many games are played altogether?

How did you work it out?

- 2** 5 schools play in a tournament. Use the same method to find how many games are played altogether. (Or change your method for a better one.)

- 3** 6 schools play in a tournament. How many games are played altogether?

- 4** What is your way of finding how many games are played by any number of schools?  
Explain your way to others in the group to check it is OK.

**Symbols and expressions**

- 1 There are  $n$  pupils in a class.  
What can  $n + 4$  mean? Make up a story.
- 2 There should be  $p$  players in a team.  
What can  $p - 3$  mean?
- 3 There were  $d$  ducks in the pond. Now there are  $d + 2$ .  
What happened?
- 4 There were  $x$  green bottles standing on a wall. Now there are  $x - 1$  green bottles.  
What happened?
- 5 You had  $b$  pounds saved. Your rich aunt keeps a promise for your birthday, and now you have  $2 \times b$  pounds.  
What did your aunt promise?
- 6 The school has  $n$  tables in the dining hall. They have to buy plastic feet for the 4 legs of each table, so as not to damage the new floor. They also buy new chairs, four for each table.  
What can  $4 \times n$  mean? (Or  $n \times 4$ ?)
- 7  $p$  is the lottery prize shared by 3 people.  
What will each get?

**8 Extension**

Make up questions like the ones on this notesheet.

Follow these steps:

- 1) Imagine a group of people or things which you have not counted and do not need to count.
- 2) Decide on a symbol to stand for how many there really are, for example  $q$  or  $y$ .
- 3) Using the symbol and numbers write some expressions (like  $q + 5$  or  $y - 4$ ).

Make up two or more stories for each expression.

# 15 Circle functions

Pupils explore the area of the circle in relation to the radius (or diameter) and then compare circumference and area as linear and quadratic functions of the radius.

Pupils work first on their intuitive understanding of the circle relationship, then verbalise and clarify their constructions, before exploring features of the graphs of these functions.

## Thinking strands

Reflection on concrete experiences of linear and quadratic functions and on their graphical representation

## Curriculum links

Area or perimeter of a square as a function of side;  $\pi$ ; Continuity

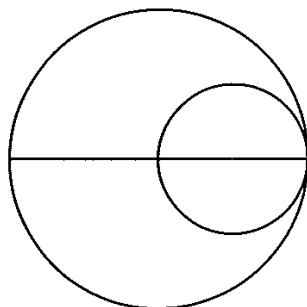
## Resources

Notesheet 1  
Notesheets 2 and 3, photocopied side by side on A3 paper if possible

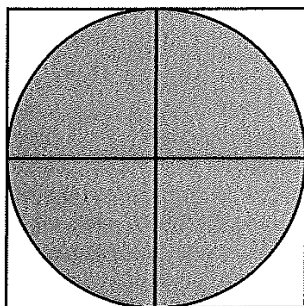
## Lesson summary

### 1 Introduction (5–10 min)

Refer to *TM14: Circle relations*, and refocus on comparisons of areas of circles. Does doubling the radius mean also doubling the area? A drawing on the board shows that is clearly wrong.



Show pupils that one way for estimating areas of circles is to complete the square on the radius, and see how much of the circle it covers.



With some classes the teacher may want to cut quarter circles out of squares and show how the trimmed parts from three squares nearly cover one of the quadrants.

Give out Notesheet 1, posing the question: "Does the same thing happen with different size circles?"

### 2 Notesheet 1 and discussion (20–25 min)

Pupils carry out for themselves work on completing the squares on the radius with a circle of different radius. You may want to have a brief class discussion after question 3, to establish that the area of a circle is slightly more than three times the area of the square on the radius.

In final discussion emphasise the two steps of calculating the square of any number and the estimation of the area. Remind pupils of the circumference to radius ratio used in *TM14: Circle relations*, and the use of  $\pi$  values as an approximation in solving problems from area to radius. Dividing by 3, then square rooting, is 'doing the opposite operations last step first', and gives a reasonable approximation.

### 3 Notesheets 2 and 3 (15–20 min)

Pose the question: "What happens to the area when the radius changes? How does that compare to what happens to the circumference?" Give out Notesheets 2 and 3.

Help pupils in drawing and joining points on the graph. Prod pupils to describe the differences between the two graphs, and encourage them to estimate mentally areas and circumferences of some circles with easy measurements.

Prepare a grid on the board to draw the two graphs and allow discussion on differences.

### 4 Class discussion (15 min)

Start after most or half the pupils have drawn the two graphs. Draw the graphs yourself or ask pupils to do so on the board. Remind the class of the similarity of the graphs to those in *TM13: Border and inside*, where one was for dots on the border, the other for dots on the inside.

Conduct a discussion on mentally estimating diameters and areas of circles with easy numbers, including the inverse operations.

## Mathematical content

This is a follow-up activity to *TM14: Circle relations*. Pupils handle the area of the circle in relation to the radius (or diameter) and then compare circumference and area as linear and quadratic functions of the radius.

Continuing the agenda of *TM14*, pupils work to ground their understanding of the circle relationship in their experiences and constructions, to reflect and to verbalise them in their own wording. That is in contrast to the algorithmic form of use in a fixed form, e.g.  $A = \pi r^2$ . Teachers are aware that such a quadratic formula is even more onerous to most pupils in the age range than the linear one for the circumference. It reinforces in pupils a view of the subject as of fixed rules and formulas on the page with the vaguest correspondence to reality. At the same time teachers and textbooks change letters and move them around seemingly at will – an alien territory.

In this lesson pupils start work on area by visually constructing a square on the radius. They are prompted to compare the area of that square to the area of a quarter circle with that radius. The quarter circle becomes an intermediate calculating unit to ease keeping the sense of size of the circle and its relation to the

radius. Pupils are led to recognise, visually and through utilising the ‘excess’ of the square on the radius to the quarter circle, how near the area of the circle is to 3 times the area of the square. The  $\pi$  ratio is to follow.

Pupils need more time to ponder this relationship, since it is more complex than the circumference to diameter comparison presentable with strings. The move from experience to expression in words should aid the move to a more mathematical presentation constructed by pairs and groups of pupils or in whole class discussion. Those temporary constructions are a prelude to accepting the algebraic formula.

The second part of the lesson focuses on handling the circumference and area as functions of a variable radius. The difference between the linear and the quadratic functions is explored in both tables and graphs and can be bridged with *TM13: Border and inside*, which deals with ideas similar to perimeter and the area of a square of variable length of side.

The last part of the lesson deals with inverse processes, whereby the radius of a circle is generated from its area, to be utilised in finding the circumference. This connects all the elements of the circle.

## Pupils' thinking

Four cognitive steps are attempted in this lesson, following the preparation in *TM14: Circle relations*. The first step is the construction through experience in the context of ‘area as a square-function’. The second step is relating that to the coverage of the circle – a multiplicative relation intuitively approaching the  $\pi$  ratio through a ‘3 and a bit’ phrase. The third step is formalising that in algebra and

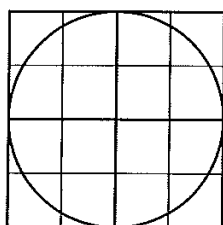
graphs. The fourth step is reflection on these new formal representations through comparison of simplified circumference and area functions, with the notion of continuity and infinity handled implicitly. Although the last of these steps may seem out of reach of most of the pupils, they can be made at an intuitive level, and should benefit all, while fully stretching the higher achievers.

## Specimen lesson

Bottom ability set, 12 pupils  
Pupils have half cm squared paper

### 0.00 Introduction

Teacher: “Draw a 4 by 4 square.”



The teacher draws the 4 by 4 grid first on the board, and then makes the central cross heavy.

“How many squares in this quarter?”

Pupil: “4.”

Teacher: “And how many have we got altogether?”

Pupil: “16.”

Teacher: “How could you find out how many squares there are in this quarter, without counting?”

Pupil: “Two times two.”

Teacher to another pupil: “And how could you find the number in the whole square?”

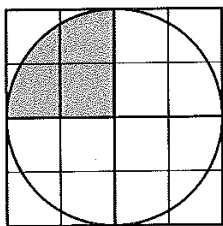
Pupil: “Four down the side, times four along the top.”

Teacher: “Using your compasses – make it two squares in radius. Where will the circle be if the point is in the middle of the square?”

Pupil: “In the big square, and touching the sides.”

### 0.08 Estimating

The teacher shades the top left quadrant of the circle on the board.



"Now I want you to estimate a bit. How many squares are in this part of the circle?"

Pupils:

"One."

"Two."

"Two and a bit."

Teacher: "And how many, then, in the whole circle?"

Pupils: various answers.

"A bit more than ten."

Teacher: "Now make a 6 by 6 square, and quarter it as before. Make the radius 3, and do a circle inside it again. What was the radius of the first circle?"

Pupil: "Two."

Teacher: "Another way of working out the area of the 3 square – how did we find the area of a quarter of the smaller square?"

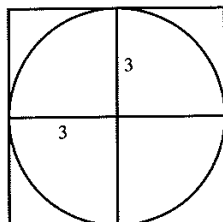
Pupil: "Two times two."

The teacher writes  $2 \times 2 \times 4$   
= 16

"So is this right?"

Pupils: "12" ... eventually "16".

The teacher makes another drawing.



Shows radius of larger circle first horizontally, then vertically

Teacher: "So what is the area of the larger square?"

Pupils: "3" ?? "3 x 3 times 6" ?

eventually "3 times 3 times 4"

The teacher writes  $3 \times 3 \times 4 = ?$

Pupils: "26" "27"

(all by trial and error, pupils have almost no multiplication table knowledge) eventually "36 – by 9 times 2, and then times 2 again"

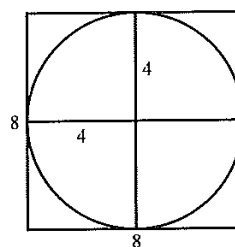
Teacher: "So what about the circle?"

(Pupils agree this is about  $4 \times 7$  or 28)

### 0.24 Another circle

Teacher: "Now we will do another one."

She draws:



Teacher: "Now, how can you get the area of the big square, starting from the quarter square?"

Pupils eventually get 64, by  $4 \times 4 = 16$ , and then twice 16 is 32, and twice 32 is 64.

The teacher now asks pupils to estimate the area of the circle.

Teacher: "How big do you estimate the circle is?"

Pupil: "Four times 4 – times 3."

Teacher: "Now do it by counting the squares as before."

Pupils: "50" "48" "49" "49"

Teacher: "And how much is that? Compare your 4 square with your 8 square – when you doubled the side, did you double the area?"

Pupils:

"No."

"It was bigger than double."

### 0.30 Tabulating

Teacher: "And how did your estimate change?"

She records:

10 28 49

She now tabulates on the board:

Radius	Estimated area of circle	Area of big square
2	10	16
3	28	36
4	49	64

Before entering the third row the teacher has on the board the numbers 50, 48, 49 and 49. She asks "What is the middle number?"

A pupil answers "48" (!)

They eventually agree it is 49.

Teacher: "Now, an extra one ..."

She adds another row

5 ? ?

into the table.

"Without drawing it, what do you think the numbers would be?"

Pupils:

"A ten by ten square."

"5 times 5 times 4" (by 25 times 2 = 50, and then 50 times 2 = 100).

Teacher: "When we were calculating the area of the total squares, we used  $4 \times 4 \times 4 = 64$ . In the three calculations for area of big squares, cross out the last times 4, and replace it with 'times 3'. How close does that get to your Estimated area of circles?"

She changes all three expressions and puts the answers in the table.

Radius	Estimated area of circle	Area of big square
2	10	(12) 16
3	28	(27) 36
4	49	(48) 64
5		(75) 100

Teacher: "Do those fit?"

Pupils: "Yes, all 3 are only a little out."

Lesson ends.

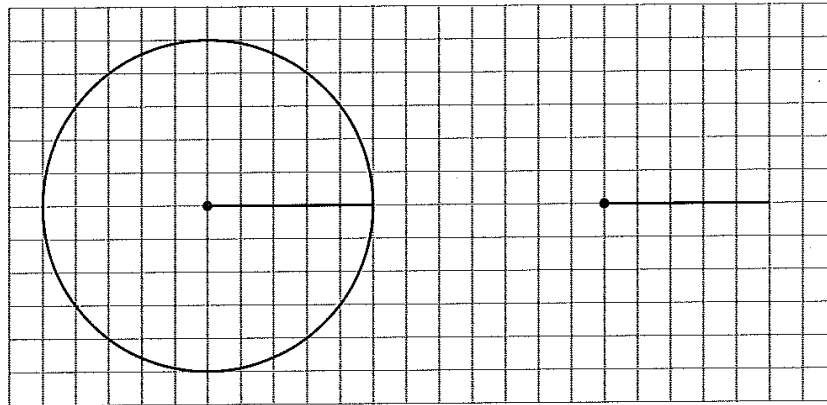


# 15 Circle functions

## Notesheet 1

### Square on the radius

- 1 The radius of this circle is 5 units.  
The radius is copied on the right.



- 2 Complete the square on the radius. Find its area.
- 3 How many of the squares on the radius do you need to cover the area of the circle?  
Choose one answer from each line that gives the best estimate:
- |           |                          |                              |
|-----------|--------------------------|------------------------------|
| (a) three | (b) three and a half     | (c) four                     |
| (a) three | (b) three and a quarter  | (c) three and three quarters |
| (a) three | (b) three and one tenths | (c) three and one third      |
- 4 Use the area of the square to estimate the area of the whole circle.
- 5 How do you find the area of any square if you know the length of one side? (Without drawing or counting.)
- 6 How can you estimate the area of any circle, if you know its radius?
- 7 Describe how the area of any circle is related to its radius.  
Use words or symbols.

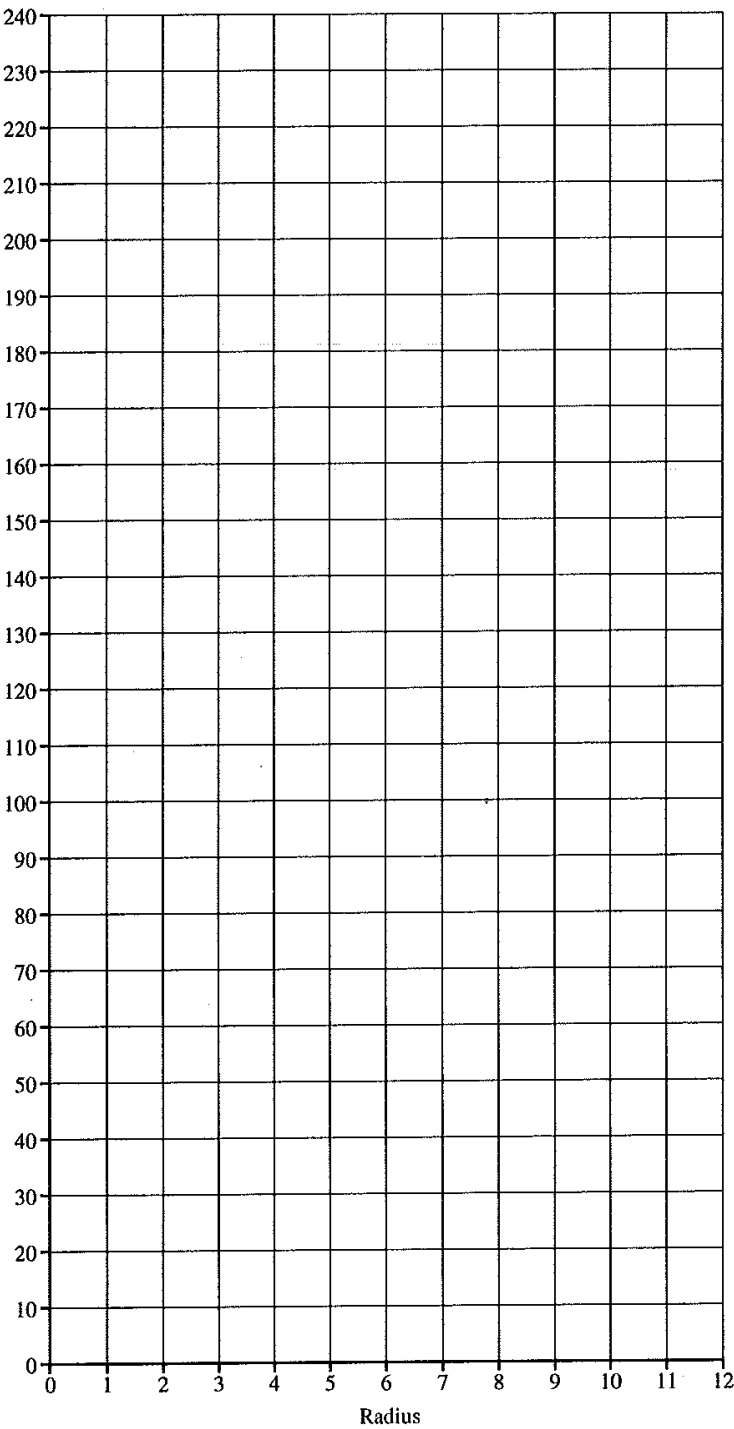
## Area and circumference

- 1 Fill in the table, taking the value of  $\pi$  as 3.  
(Or use its value in the calculator and then round to nearest whole number.)

Radius	Circumference	Area
1		
2		
3		
4		
5		
6		
7		
8		
9		

- 2 Draw the graphs for the circumference and the area on the grid on Notesheet 3.  
Use letter A for area points and letter C for circumference points.  
Join the letters.
- 3 Describe the differences between the two graphs.
- 4 A circle has an area about 100.  
(a) How do you find its radius?  
  
(b) How do you find its circumference?
- 5 A circle has area about 1000 (one thousand).  
How do you find the radius?

Use × for Area.      Use • for circumference



## 20 Heads or tails?

Pupils record the number of heads they get in five samples of 10 tosses of a coin. They contrast the variation in the percentage of heads in their samples of 10 with the way the percentage approaches more and more closely to 50% as the class data is cumulated. This introduces the idea of sampling variation.

Then they look at the pattern of the frequency of runs of heads in the whole class data. The frequency roughly halves between runs of 1 and runs of 2, and continues to halve with each longer run. They discuss this in terms of probability.

### Thinking strands

Models of probability and chance

### Curriculum links

Data-handling: probability and proportions

### Resources

Supply of two-pence pieces  
Notesheets 1 and 2 for each pair of pupils  
Table 1 for whole class cumulative proportions copied on to A3 or A2 format  
Chart 1 copied on to A3 or A2, landscape format, for plotting cumulative proportions  
Table 2 and Chart 2 for frequency of runs of heads – A3 or A2 format  
Wide felt tip marker

## Lesson summary

### 1 Introduction (5 min)

Introduce the activity as testing the expectation that there is an equal chance of getting heads and tails when throwing (tossing) a coin. Have a class demonstration with a sample of 10 throws. Will that produce 5 heads? Discuss fairness in dice and coins, perhaps by asking pupils to demonstrate unfair and fair throws, so they understand how to avoid bias in the experiments they will do on Notesheet 1.

### 2 Notesheet 1 (15 min)

Introduce Notesheet 1. Give notice that in question 4 they will have to convert 'something out of 10' into a decimal and percentage. Show and leave examples on the board for scores such as '3 out of 10' and '21 out of 50'.

Pupils should work in pairs. Some pairs may need to be shown how to plot their proportions into the graph in question 5.

While the class is working prepare Table 1, the cumulative table of heads, on the board. You may prefer to prepare it beforehand enlarged on A3 paper or on OHT. Similarly prepare the related cumulative graph, Chart 1.

Start the discussion on class results when some ideas arise in question 6, on why their total sample (50) variation is smaller than for samples of 10. All the pairs should check to see if this is true for their samples, appreciating that they need to use a percentage or a decimal proportion to do that, while also referring to the raw scores for their samples of 10 and question 2.

### 3 Cumulative table and chart and discussion (15 min)

Now fill in the results in the cumulative table of heads (Table 1). Have pairs who have extreme results in one direction, e.g. nearer to 30%, to be the first to place their results in the table, in order to highlight the expected convergence of the cumulative results towards 50% with the increase of sample size. You may want to indicate to the left of each sample of 50 to which pair it belongs.

As the class are writing in their results you can fill in the cumulative column for number of heads, total of heads so far, and proportion. Then plot the values one by one from the table on to the cumulative chart (Chart 1), illustrating the one-to-one relationship between the table and the chart. (You may want to emphasise the bigger variations in the samples of 10 by plotting on the graph the proportions in those samples of the first two pairs of pupils.) Involve the class in calculating the ratio in each step and in checking how the proportion is behaving: is it getting nearer or further away from the hypothetical 50%? Repeat until all the pairs have given their results.

How do pupils know that the proportion is really getting nearer to the half and not just continuously fluctuating? What sample size is enough to convince us that the fluctuation is getting smaller? Ask what the variables in the graph are (sample size and cumulative proportion of heads). If the mathematicians claim that the limit value of the proportion is exactly 0.5, how do they interpret this? Mention the terms 'frequency', 'relative' and 'limit'.

#### 4 Notesheet 2 questions 1 and 2 (5 min)

Give out Notesheet 2. Ask pupils: "If you get 5 heads in a row, is the chance that the next toss is a head more than if we start again?" Explain that questions 1 and 2 are there to help them in the process of gathering the whole class's data on the question, so that they do that quickly to give the results to the teacher. You may need to explain what is meant by a 'run', and that the runs can continue across the dividing lines between the 10's on Notesheet 1.

#### 5 Frequency of runs: Table 2 and Chart 2 (15 min)

Collate the whole class data in Table 2 as quickly as possible, and then show the whole class how the pattern develops in Chart 2 (use a fat felt marker to dramatise the pattern). You can either fill the chart at the same time as you write in the table, or wait to find the totals for all the class.

The pupils should now discuss the class results as shown on Chart 2 with their partner(s), following the order of questions 3 to 7 on Notesheet 2.

#### 6 Class discussion on patterns of runs (10 min)

The agenda here is to move from general recognition that there is a decrease in the heights of the columns as the run increases to the pattern of the decrease, then to quantifying the decrease as approximately halving each step. Use the words 'roughly', 'nearly', 'almost', the difference between 'About how many times' and 'How much bigger'.

Why does the pattern look like a halving one? What is the shape of the halving pattern (an exponential decrease)? Do past actions affect future ones? In what sense 'yes'? In what sense 'no'? Leave it hanging!

### Background notes

The following notes give more detail on the agenda covered by the first part of the lesson. The data used has been randomly generated by a simulation on the computer.

For Notesheet 1 a discussion on the sampling variation can be conducted in a number of ways. One way is to have on the board Table A showing the variations of the proportion of heads in the 10 throws samples and in the 50 throws sample for each pair or group.

The 10 throws samples		The 50 throws sample
Least	Most	
0.3	0.7	0.56
0.4	0.8	0.60
0.2	0.6	0.46
0.4	0.6	0.57
0.4	0.6	0.45

Variation in the samples of 10 throws compared to the variations in the sample of 50.

So the variation in the 10 throws samples is between 0.2 and 0.8. In the 50 throws sample the variation is between 0.45 and 0.60. For some classes the numbers can be truncated or rounded to one decimal place only, with the difference qualitatively noted as 'a bit bigger', or 'fair way bigger'.

With a high attaining class, for whom the practicalities of the activity are straightforward, you might handle the discussion of Notesheet 1 on the move to the cumulative sample in the following way, introducing a more interesting real life context, that of the gender of the newly born child:

"In the case of a coin we can see that the sides are symmetrical in weight, etc. and so we expect

the number of heads and tails to be equal, if the coin is a fair one. But suppose we are just starting from the results we get from small samples, and want to use the results to find out what the chances really are of the two possible outcomes?

"For example, we know that a baby can be a boy or a girl. But are there some causes in the process of conception or peculiar to the area or a historical period which may give a bias favouring more boys than girls or vice versa? How many babies would we need to count in a hospital's records before we decided that there is or is not such a bias?

"We can investigate by analysing data in each batch of 10 births separately, as in Graph A."

Alternatively we can add the batches to make a much larger sample, where the ratio is repeatedly recalculated as in Graph B.

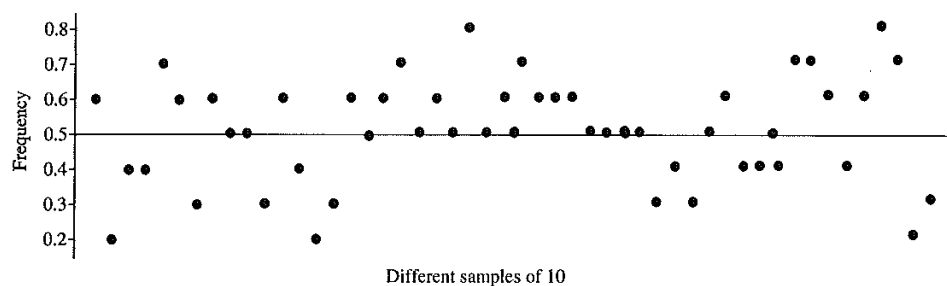
"Discuss: to the nearest 100, how large a sample of births would you need to collect from a hospital before you could decide if there was a bias in favour of boys?"

Basically they have to see, in some way, that something like 500 or more births are going to be needed, by contrasting Graph A with Graph B.

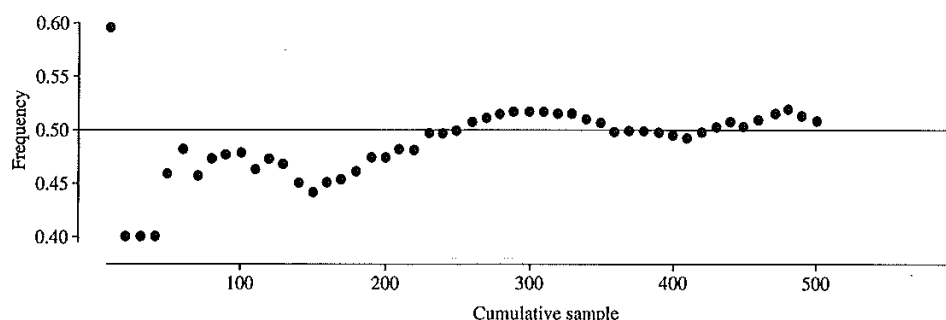
A fruitful discussion may follow a 'second go' at using the samples of 50 throws (births) in a different order. Suppose that your first samples accidentally are all lower than the half way, then the point of reaching a reasonable closeness to the half-way mark is much later!

Another approach is to bridge the activity to the previous work on probability.

"Do you remember, in the *TM16: Three dice* lesson, how much variation there was between the numbers in the samples you took there? The technical name for this is sampling variation."



Graph A Range of variations in the proportion of boy babies born for all the samples of 10 babies.



Graph B Range of variation decreases with cumulatively larger sample of babies born.

## Mathematical content

This activity is designed to engage pupils in four basic ideas about probability.

- 1 You can treat the probability of an event in terms of a decimal fraction between 0 and 1 and/or percentage between 0 and 100%.
- 2 By implication, there are two approaches to probability. The first is causal, where the causality is known in advance. Since we know that coins are symmetrical, we have a prior expectation that the probability of throwing a head should be 0.5. The second is observational (experimental probability). We actually count the occurrence of the possibilities (two alternatives in this case), and work out the probability from the sample of observations made. But from a small sample the proportions may vary wildly from what we expect.
- 3 The idea that individual events do not depend on the previous history. This is investigated by counting the runs of heads,

and thinking about the fact that the observed probability of a run halves each time we increase the number of heads by one.

- 4 If we make the sample large enough, then the observed probability eventually approaches the causal probability (or in statistical terms, approaches the probability as estimated from analysis of the probability space). The point here for the pupils is how large the number of observations has to be: well in excess of 200, and for safety closer to 500 or 600. In this way we prefigure the notion of estimation – to be developed much later in instructional lessons.

Some of these ideas have been touched on, in a qualitative way, in *TM16: Three dice*, but here the relation between theoretical and experimental probability is made explicit and quantitative. Pupils will get the chance to encounter the four ideas involved here again in *TM24: Data relations*.

## **Pupils' thinking**

Probability is one of the ten 'formal operational schemata' or reasoning patterns which Piaget described as characterising the formal stage. It goes beyond concrete operational thinking in two ways: first, it involves comparing ratios so as to look for the underlying proportions determining the probability. Secondly, it involves making some kind of mental model of reality, and then seeing to what extent reality confirms or behaves in accord with the model.

So the intervention aspect of this lesson is to design an investigation which allows each pupil to go as far into these two aspects as he/she is capable of at the time, without treating the lesson in instructional terms as having a single learning agenda for all the class.

Other lessons have featured other reasoning patterns, so here we are extending the variety for the pupils, on the grounds that the more reasoning patterns they encounter, the higher is the chance that the experiences reinforce each other.

**Throwing coins**

Spin or throw your coin each time.

Write **H** if it comes down heads, **T** if it comes down tails.

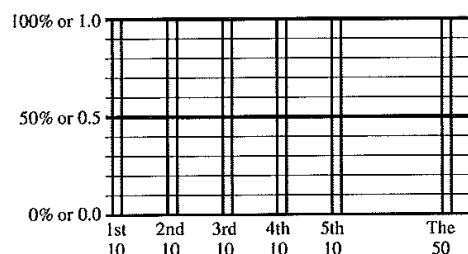
For each group of 10 throws, write down how many heads came down.

Complete all the 50 throws before answering the questions below.

If your coin is a fair coin, there should be **equal chance** of getting heads and tails.

To check that, we take each group of 10 throws as a **sample**.

- Did you get exactly 5 heads in any of your samples of 10 throws? \_\_\_\_\_
- What was the **largest** number of heads you got in any sample of 10 throws? \_\_\_\_\_  
The **smallest** number? \_\_\_\_\_
- Now take all the 50 throws as a sample.  
How many heads in 50 throws? \_\_\_\_\_
- To compare the proportion of heads in different size samples, we use percentages  
(between 0% and 100%) or parts of 1  
(between 0.0 and 1.0) in each sample.  
Fill these in:  
10 out of 10 is 100% or whole one (1.0).  
5 out of 10 is \_\_\_\_% or half of 1 (\_\_\_\_)  
1 out of 10 is \_\_\_\_% or one tenth (\_\_\_\_)  
50 out of 50 is 100% or whole one (1.0).  
25 out of 50 is \_\_\_\_% or half of 1 (\_\_\_\_)  
5 out of 50 is \_\_\_\_% or one tenth (\_\_\_\_)
- Mark the proportions of heads for each of your samples of 10 on the scale.  
Draw the proportion of heads in your sample of 50.



Throw	H or T	
1		
2		
3		
4		
5		
6		
7		First 10
8		Number of heads
9		_____
10		
11		
12		
13		
14		
15		
16		Second 10
17		Number of heads
18		_____
19		
20		
21		
22		
23		
24		
25		
26		Third 10
27		Number of heads
28		_____
29		
30		
31		
32		
33		
34		
35		
36		Fourth 10
37		Number of heads
38		_____
39		
40		
41		
42		
43		
44		
45		
46		Fifth 10
47		Number of heads
48		_____
49		
50		

- Is the proportion of heads in the 50 throws sample nearer to the 50% (0.5) mark than in some of the samples of 10? \_\_\_\_\_  
Explain.



**Runs of heads**

- 1 Go through your table in Notesheet 1 looking at runs of Heads, for all 50 tosses..  
What is the longest run of Heads you have? \_\_\_\_\_
- 2 Count the frequency of runs of one Head.  
Then count the frequency of runs of two Heads, and so on. Fill this frequency table:

Runs of:	Frequency
1 Head	
2 Heads	
3 Heads	
4 Heads	
5 Heads	
6 Heads	
7 Heads	
8 Heads	
9 Heads	

Examples:

... T H T ... is a run of 1 Head... T H H T ... is a run of 2 Heads... T H H H T ... is a run of 3 Heads

Look at Chart 2.

Discuss these questions with your partners, so you can tell your ideas to the rest of the class:

- 3 Is there a pattern in the runs of heads?
- 4 Roughly, about how many times more runs of **one head** were there than runs of **two heads**?
- 5 Roughly, about how many times more runs of **two heads** were there than runs of **three heads**?
- 6 Suppose you have thrown two heads in a row, and now do another fair throw.  
What do you think the chances are this time of getting a Head?
- 7 Now look again at Chart 2.  
Give a description of the pattern.

Cumulative table for proportions of heads

Number in sample	No. of heads in sample	Cumulative number	Total of heads so far	Proportion of heads so far
10		10		
10		20		
10		30		
10		40		
10		50		
10		60		
10		70		
10		80		
10		90		
10		100		
50		150		
50		200		
50		250		
50		300		
50		350		
50		400		
50		450		
50		500		
50		550		
50		600		
50		650		
50		700		
50		750		
50		800		
				(Total of heads so far ÷ Cumulative number)

## Heads or tails?

Cumulative proportion of heads as sample size increases

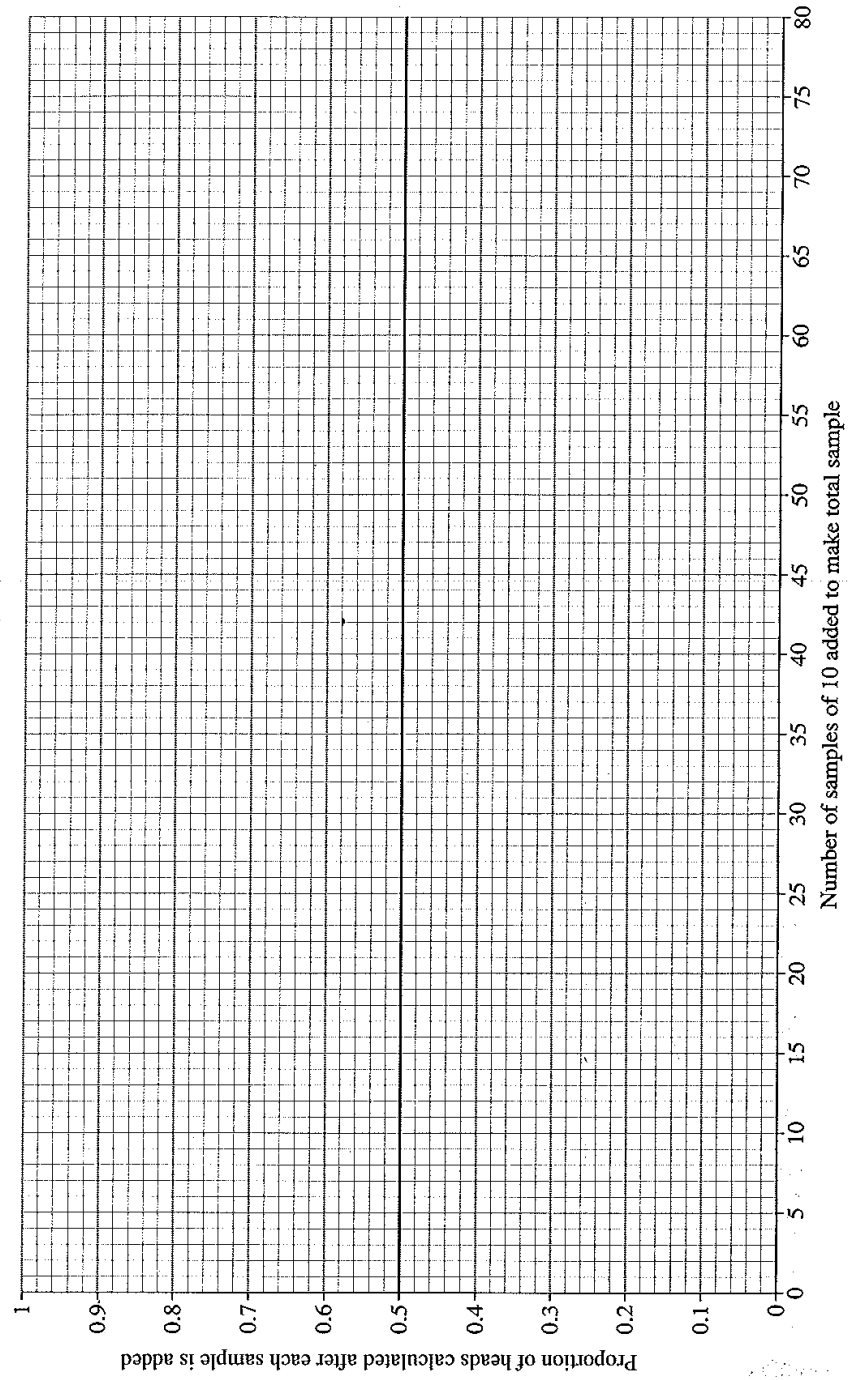
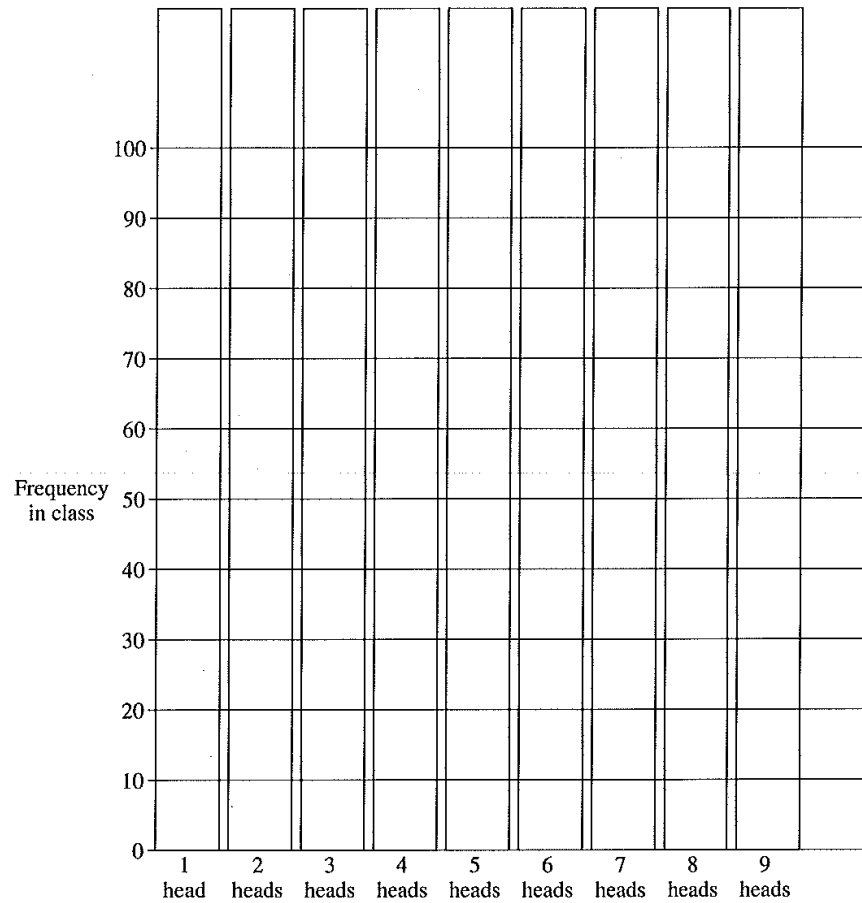


Chart 1

### Cumulative table for frequency of runs

[illegible]

As the run of heads gets longer ...



# 22

## Comparing correlations

This is a follow-up to *TM18: Correlation scatters*. The emphasis is on moving from the intuitive notion that a wide scatter means a poor correlation to comparing correlations quantitatively. Three different data-sets are compared by reducing them to four-cell tables of confirming and disconfirming cases. Correlation is estimated as a ratio of confirming to disconfirming cases.

### Thinking strands

Correlation as the degree of fit between one variable and another

### Curriculum links

Data-handling: median and range; estimation and precision

### Resources

Notesheets 1 and 2  
Graphs 1, 2 and 3 enlarged to A3 for use on the board  
Rulers

## Lesson summary

### 1 Introduction (10 min)

Have the three enlarged graphs ready to pin on the board.

Ask pupils for their ideas on how the grades they might get in Science may compare with their grades in Maths. Then, using the Science/Maths Graph 1 at the board, talk them through, by asking questions, everything that they need to do to produce a line of best fit (using medians at each grade), reminding them of what they did in *TM18: Correlation scatters*. Draw in a line of best fit using a coloured felt pen, then ask the pupils how useful that is in predicting from Maths to Science. They should think of the range of predictions for a given Maths grade as the vertical range that corresponds to it in the Science grades. You may want to draw pupils' attention to the overall oval shape of the data and point out the width against the length as corresponding to the range of prediction vs the total range of possible values. Give out Notesheet 1.

### 2 Notesheet 1 (20 min)

Pupils should draw the lines of best fit on the graphs. Depending on the ability range of the class, pupils could either

- be asked to work on all three of the graphs on Notesheet 1 in pairs first, followed by a discussion of enlarged versions of each graph pinned to the board, or
- have short sessions at the board after they work on each in turn.

Whichever strategy is used, class discussion at the end should finish with comparing each in terms of which graph gives the best, middle and worst prediction, and then in terms of the range of prediction in each case.

Get the pupils to quantify the ranges in approximate terms, e.g. that for the 100 m and

200 m sprints the range of the estimates is about 0.2 seconds while the total range of results is about 0.8 seconds. They should compare that to the Maths/Science ratio of 4 to 7 grades, and to the English/PE ratio of 50 to 50 marks. They should see that these ratios correspond to the 'narrow oval', the 'middle oval' and the 'near circular' overall shapes of the data on the graphs.

### 3 Introduction to Notesheet 2 (15 min)

Talk through the Science/Maths graph at the board, showing pupils how to make two median splits. For the first split, count up from grade G, vertically for science and horizontally for maths. This takes you to just above E for science (so the split includes the 11<sup>th</sup> cross and has 12 cases below the line) and just to the right of D for maths (so the split includes the 11<sup>th</sup> cross and has 13 cases to the left of the line). Note that this would be different if we were to 'count down' from highest grades. The four cells can be called 'high/high; low/low; high /low, and low/high'.

low/high 3	high/high 7
low/low 10	high/low 2

Then talk through the numbers in terms of high/high (7) and low/low (10) confirming the relation, and the other as disconfirming. Ask different pupils to explain the meaning of this pairing of cells in their own terms. Then ask how to quantify the relationship between all confirming cases to all disconfirming cases. Expect answers of the type: "there are 17 confirming cases, which is 12 more than the 5 disconfirming cases", or more sophisticated answers of the type "there are about 3 confirming cases to each disconfirming case".

#### 4 Notesheet 2 and class discussion (15 min)

Allow time for the class to work through the other two ratios on the Notesheet. Then conduct another exchange of ideas about good, bad and fair prediction and about the notion of correlation. It may help to draw in, with a felt marker, narrow and wide ellipses around the scatters for runners and Science/Maths grades, and a rough circle for English/PE.

Higher attaining pupils may be able to link the ratio of confirming to disconfirming cases to the visual ratio of width and length of ellipses, and to the ratio of the range of prediction to the total range. They could see the linear relation as the limit of the correlation relationship. But there is no need to proceed to formal values for correlation, which relate all values to the line of best fit and measure how good that is.

### Mathematical content

In *TM18: Correlation scatters* pupils will have developed some ideas about how to look at a correlation scatter in terms of predicting from one variable to another, and about the degree of uncertainty of a prediction. Here we ask them to look at a correlation in terms of its strength. The two sets of ideas are related by realising that the lower the degree of uncertainty (the range of a prediction) the greater the correlation.

There is a rather large hierarchical step between intuitive concepts of correlation – processed by looking at the degree of scatter between the two variables concerned – and quantitative concepts intended to provide a measure of correlation. Our problem as interventionists with one eye on the National Curriculum is to make our treatment of correlation at a level where most 12-year-old pupils can handle it ‘in some intellectually honest form’ consistent with the underlying principles of coefficients of correlation.

There is a whole family of quantitative measures of correlation which begin with either the full correlation matrix, or the same matrix reduced to a four-cell table:

	variable Y		
	0	1	
(a + b)	a	b	1
(c + d)	c	d	0
	(a + c)	(b + d)	variable X

In the four cells the ‘1’s and the ‘0’s refer either to success or failure, or to high and low values of the two variables concerned, often obtained by dichotomising the values at the median. So cells *b* and *c* refer to cases tending to confirm a correlation relation, and cells *a* and *d* the cases tending to disconfirm the correlation. If cells *a* and *d* are empty or nearly empty, we obtain a very large number, and the correlation is nearly perfect.

Some measures of association have in common the expression  $(bc - ad)$  in the numerator, and then some function of *a*, *b*, *c* and *d* in the denominator, chosen in order to make the total expression vary between 0 and 1. For example, Pearson’s *r*, for a 4-cell table is given by

$$\frac{(bc - ad)}{\sqrt{(a + b)(c + d)(a + c)(b + d)}}$$

This allows *r* to vary between 1 (perfect correlation), 0 (no correlation) and –1 (negative correlation).

The solution suggested here is to use a much simpler ratio,  $(b + c)/(a + d)$ , the sum of confirming to the sum of disconfirming cases – as used in computing tetrachoric correlation. It also agrees with intuitive judgements about relative degrees of correlation in that, when the sum of confirming and dis-confirming cases is the same, there is obviously no correlation; the ratio is 1, and the correlation is 0.

This is handled by giving pupils three different correlation scatters to inspect. First they look at them in terms of the range of prediction, which relates back to *TM18: Correlation scatters*, and then they look at the same scatters by reducing them to four-cell tables, and counting the confirming and disconfirming cases.

### Pupils’ thinking

The National Curriculum has at Level 6 ‘... have a basic understanding of correlation’, and both at Key Stage 3 and GCSE pupils are given four-cell tables of numbers and asked questions about them. The problem with realising the National Curriculum aim, from a

developmental point of view, is that correlation is a mental model which does not describe reality directly, but on which one has to do further work to connect the data one collects to inferences drawn from the model (see notes on *TM18: Correlation scatters*). We are not thinking

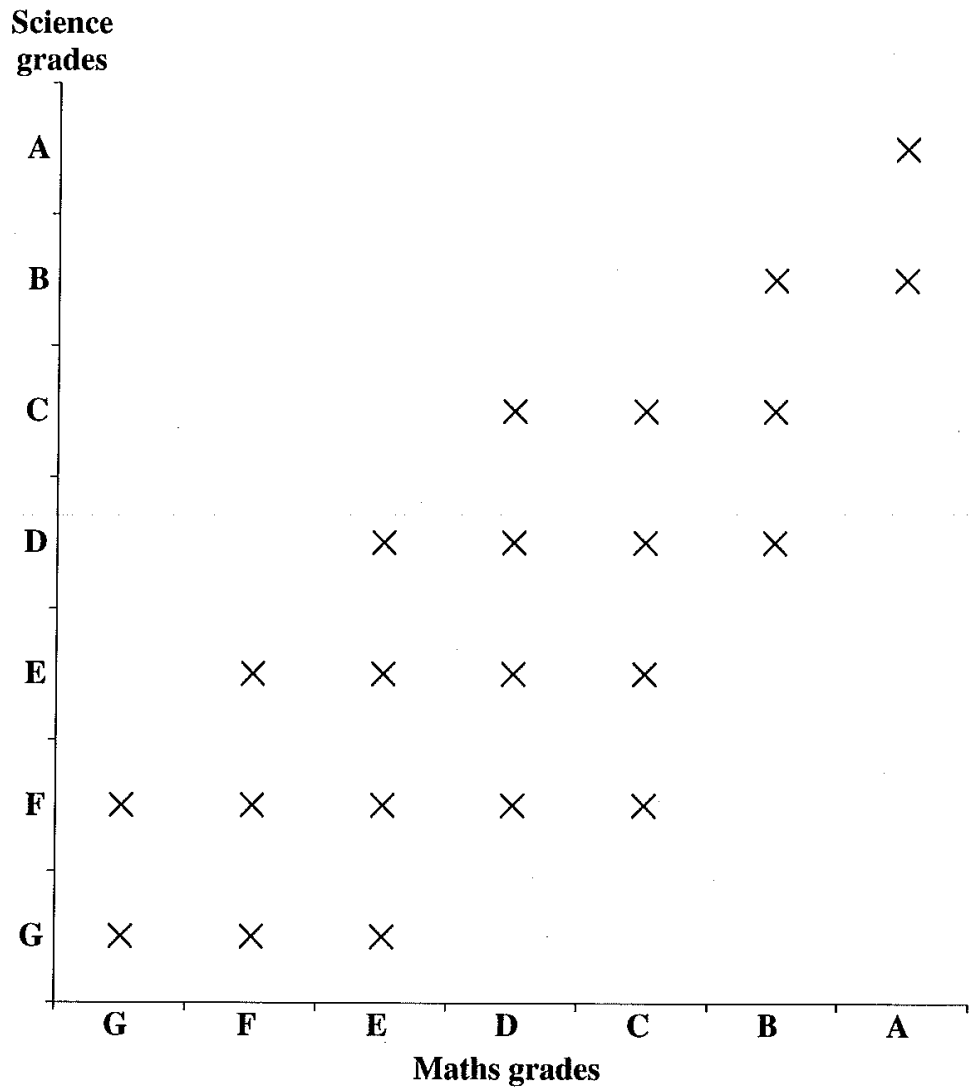
here of anything formal and complex, like calculating Pearson correlation coefficients or standard deviations or the like. It is just that the degree of a relationship, however you come at it, is a hypothesis about reality which you have to think of ways to test.

A pupil who is given a four-cell table of data on two variables has to have some ideas to use in order to compare one set of figures with another. Our suggestion of

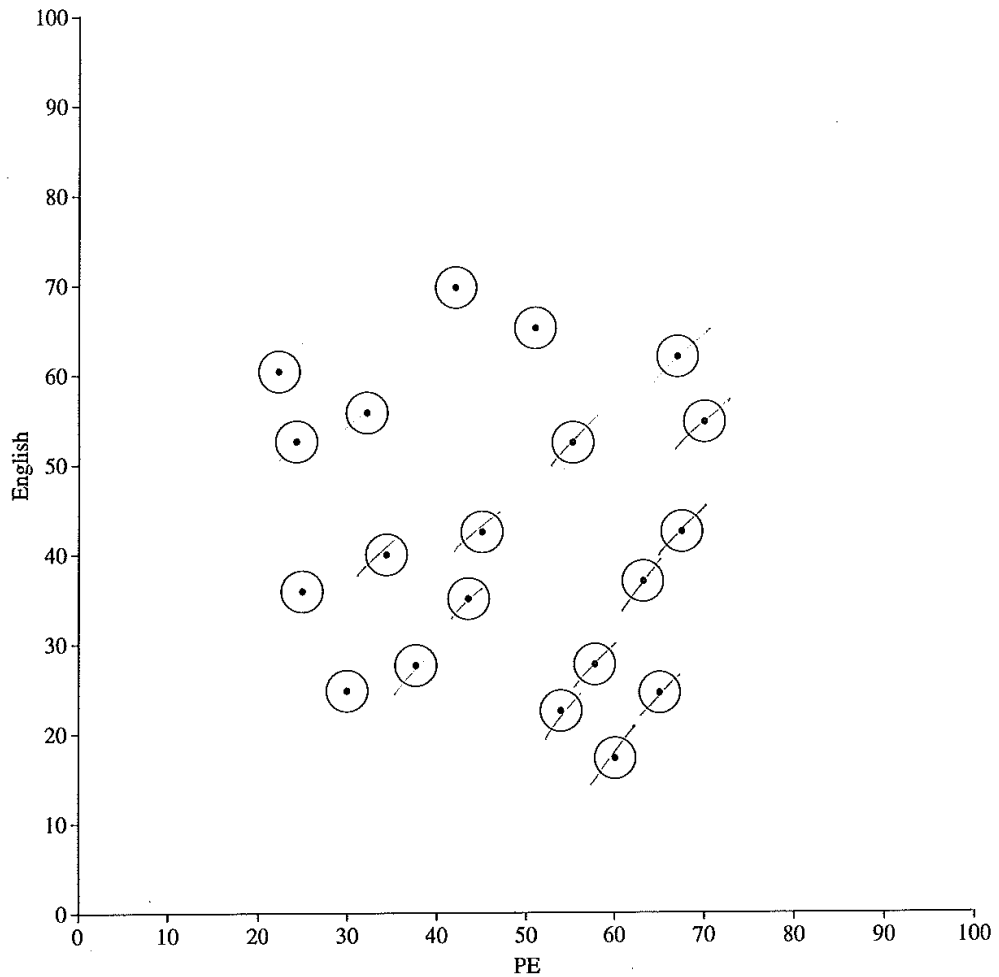
counting the number of confirming and disconfirming cases and then calculate their ratio is the lowest level algorithm we can think of which is still intellectually honest. Different pupils will make different use of it, but the intention is to provide something which will help the more able pupils to think further about what correlation means, while giving the less able pupils something which they can handle.



Science and Maths grades



English and PE marks

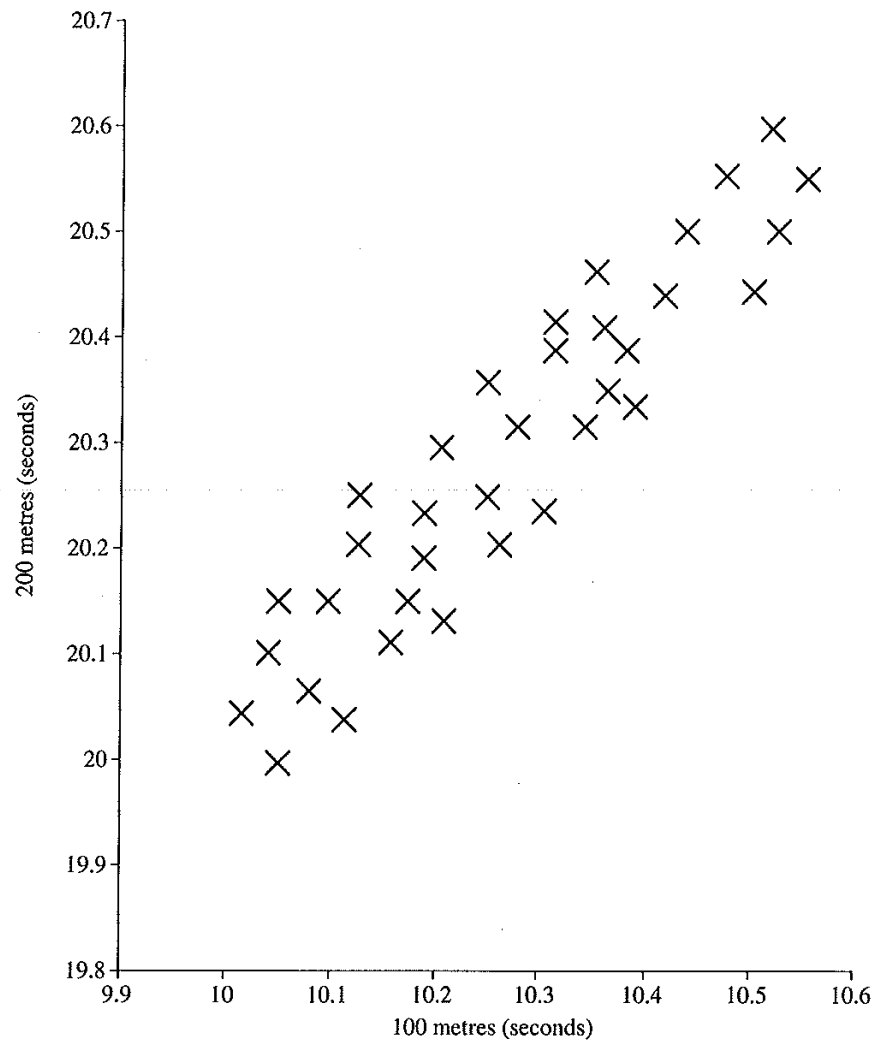


# 22

## Comparing correlations

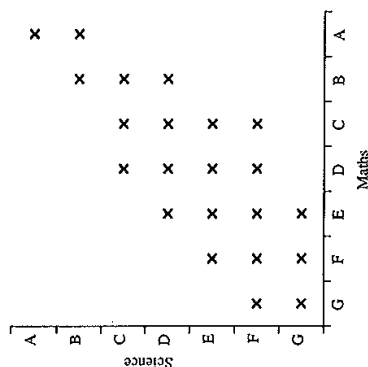
Graph 3

Times for 200 m and 100 m sprints



## Good, fair, or useless prediction?

Maths and Science grades — 22 girls



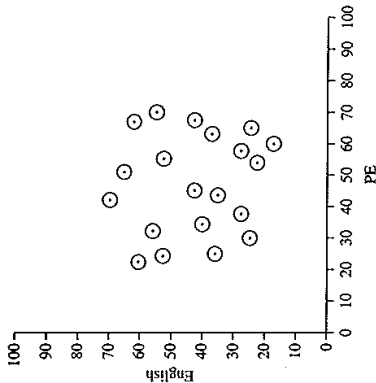
For each Maths grade find the middle (median) Science grade. Then draw the line of best prediction (best fit).

For a maths grade E, C or B, what is the range of Science grades possible?

What is the best prediction?

Compare the range for each grade with the total range. Do you think the prediction is useless, OK or quite good?

Marks for English and PE — 20 boys

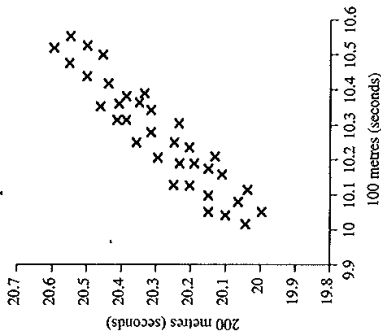


For every group of PE marks find the median English mark. Draw the line of best prediction.

For any PE mark, what is the range of English marks, and the best prediction mark?

Compare the range with the total range. How good is the prediction in this graph?

Times for 100 m and 200 m sprints — 35 mixed



Draw the line of best prediction.

For some examples of times for 100 m, find the range and the best predictions.

How good is prediction in this graph?

Looking at the three examples of correlation above, describe what makes:

- A good prediction:
- A bad prediction:
- A fair prediction:

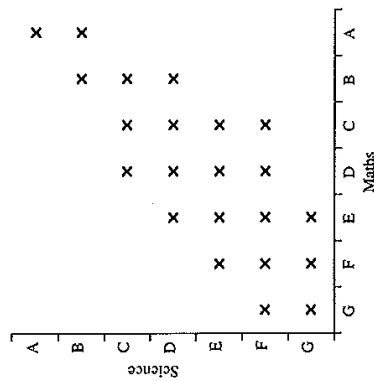
# 22

## Comparing correlations

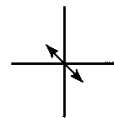
### Notesheet 2

#### Measuring correlation

Maths and Science grades — 22 girls

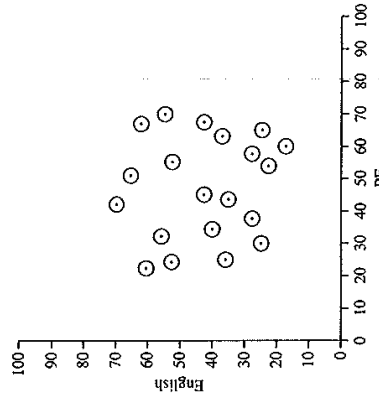


Split the data in halves vertically with a line (by Maths grades).  
Split the data in halves horizontally (by Science grades).  
Count and fill in the 4-cell table:

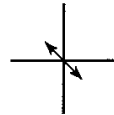


The number of confirming grades is: \_\_\_\_\_  
The number of disconfirming grades is: \_\_\_\_\_  
The ratio of confirming to disconfirming is about: \_\_\_\_\_

Marks for English and PE — 20 boys

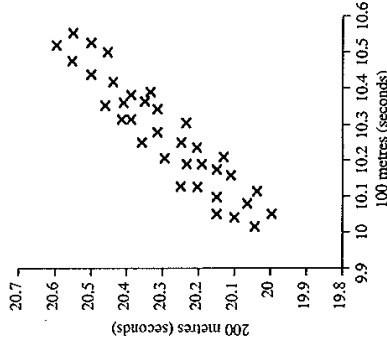


Split the data vertically and horizontally. Fill in the table.

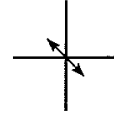


The number of confirming grades is: \_\_\_\_\_  
The number of disconfirming grades is: \_\_\_\_\_  
The ratio of confirming to disconfirming is about: \_\_\_\_\_

Times for 100 m and 200 m sprints — 35 mixed



Split the data vertically and horizontally. Fill in the table.



The number of confirming grades is: \_\_\_\_\_  
The number of disconfirming grades is: \_\_\_\_\_  
The ratio of confirming to disconfirming is about: \_\_\_\_\_

Look at the numerical ratios of confirming to disconfirming cases. What ratio is about right for:  
Good correlation and prediction?  
Bad correlation and prediction?  
Fair correlation and prediction?

Pupils contrast simple acceleration, as met in *TM23: Rates of change*, with exponential growth, using tables of values at unit time intervals. By inserting one extra column they can see the pattern of increase in unit time (constant for the linear relation, and steadily increasing for the quadratic). But with the exponential even the rate of increase of the increase goes on increasing. Here the emphasis is on the real life implications of these different mathematical models.

#### Thinking strands

Multiplicative relations; metacognition and bridging

#### Curriculum links

Linear, quadratic and exponential functions in mathematics.

Constant acceleration in physics, exponential growth in biology, explosive reactions in chemistry.

#### Resources

Notesheet 1; Notesheets 2A and 2B copied on to one A3 sheet)

The graph from Notesheet 1, enlarged  
Calculators with 10 digit display

### Lesson summary

#### 1 Introduction to Notesheet 1 (10–15 min)

Remind pupils of the work they did in *TM23: Rates of change*, showing them the large version of the graph. Give out Notesheet 1 and pose the question: “Describe in words the difference between the rising of the two different craft, using the words ‘speed’ and ‘acceleration’”. When you double the time, in the two cases, how many times does the height they have risen increase?” Give pupils about 5 minutes to think about their answers.

In class discussion mode, keep asking different groups to contribute until you feel that they are using terms such as speed and acceleration in a way that describes the two graphs. Here we are not after mathematical accuracy, rather they use everyday words to describe these things in both mathematics and science. For example, for the case of the balloon where the speed stays the same: “It means every second they go the same distance”, “The graph keeps the same slope”, “stays a straight line”, “The table shows change of height in equal steps”, “like a staircase with all the risers and treads the same”.

For the case of the steady acceleration of the rocket, accept explanations such as: “It starts slower than the balloon but the change of height gets bigger and bigger”, “It makes up for the slow rise in 8 seconds”, “If you double the time, the rise per second doubles, while with the balloon the rise doesn’t change”.

Pupils may recognise the square function, and you need to get them to clarify what

variable is the square of what other variable, i.e. to see the relationship not simply as a sequence, but that this relationship applies for different values of time not noted in the table. For example, at time = 40, the height would be 1600 m.

#### 2 Introduction to Notesheets 2A and 2B (10 min)

Start with an example of growth where something doubles each time: “There is another kind of acceleration. A baby starts as a single cell: then this cell splits into two cells. Then each new cell becomes two again, and so on. It is the same when you start with a little yeast in a bread mixture, and you wait for it to start to rise fast. Each of the yeast particles splits into two, then each of those splits into two, time after time. You can find out about this other kind of acceleration by working this ancient problem.

“In Babylon a General had conducted a famous victory over an invading army. The King asked him to name his reward. He called for a chess board and said, ‘Just give me one grain of wheat, on the first square, 2 grains on the second square, 4 grains on the third – doubling each time – until all the squares on the board are full.’

“‘You aren’t asking for much’, said the King. ‘You can certainly have that.’ But one of the Court astronomers thought a while, and did some calculations on a wax tablet, and then said, ‘You may regret this!’

“Why do you think he said that?”

Talk through a way of imagining big numbers with the pupils. Write a summary of the conversion values of kg/tonne on the board:

“When I weigh 100 grams of wheat grains on a Post Office scale, then count them, the number would be about 200 grains. So

1 kilogram weighs 1000 g. That’s 10 lots of 100 g. That’s 2000 grains, 10 lots of 200 grains.

There are 1000 kilograms in one tonne. So each tonne of grains must contain 1000 times 2000 grains, or 2 million (2 000 000 grains). A big lorry carries about 50 tonnes, so about 100 tonnes fills a classroom!”

### 3 Notesheets 2A and 2B (15 min)

Pupils work in groups, using calculators, on questions 1 and 2. Encourage them to see the pattern of doubling the number of grains in the middle column. As they are working, go around asking the groups if they can think of other ways of solving question 2. They should notice the advantage of using  $2^{10}$  as a convenient unit in its own right. ( $2^{10}$  – 10 steps of doubling – gives an increase of 1024, which can be rounded to 1000 to the nearest two decimal places, with a small error of about 2%.)

(Note: the large numbers may cause some problems even with calculators. Arithmetic calculators are preferred – to avoid scientific notation. Even if the display is less than 10 digits, pupils can still think about how to keep track of the zeros – chunking using times 10, 100, 1000 etc. – for the rounded or approximate values.)

### 4 Class discussion (15 min)

When most of the class have made some progress in questions 1 and 2, stop for general discussion. Ask them to summarise their

conclusions, and deal with any computational difficulties.

Question 2 should not be given much time, as the main focus of the lesson is on comparing the functions. One of the difficulties in the question is that the 30th square takes about 1 000 000 000 grains (not the two thousand million needed), but you just need to remember that the sum is cumulative and this much has already been used on the earlier squares.

Use the poster version of the first graph on the board with vertical axis extended to, say, 500 (for  $2^9$ ) and have the pupils imagine  $2^{10}$ , and sketch in a different colour the pattern of increase of the number of grains against the square number. Now compare this new graph with the linear and quadratic patterns already there.

Show pupils if necessary how to fill in columns 4 and 5 of their table, and give them a few minutes to finish this and discuss with their partners how to describe this new type of acceleration in words and numbers, and how it might connect up with natural events the teacher knows about. You may use explosives as an example. A small detonator is first triggered, which triggers 3 or 4 others, each of which triggers 3 or 4 more, etc., all within a fraction of a second.

General class discussion should feature both the maths (“What is the difference between the rocket acceleration and the grain acceleration? Can you put it into words?”), and also the real life applications (starting just with one fertilised cell how would a complete baby ever be produced unless acceleration could be accelerated? – but this might also bring up the issue that there must be some kind of growth termination going on also, otherwise ...!).

## Mathematical content

The intention is to extend the agenda of *TM23: Rates of change* from linear and quadratic relations to their first acquaintance with the weird properties of an exponential. It is not intended as an introduction to the mathematics of exponentials, but rather as one further link to the mathematical modelling which is used, without explanation or background, in science.

Here we want them to see that there are two different kinds of acceleration. The first, which in *TM23* was applied to the motion of a rocket, is the kind they will get in Physics (constant acceleration: e.g. in Year 8 they might be asked to use sensors to capture data for an accelerating trolley, and note the

increasing distance between the dots on the tape, and make some kind of graph or sense of it). But the second is more usually met in Biology (e.g. the growth of yeast, or the way in which a cold virus grows in the body from a tiny beginning in someone’s sneeze, or even the growth of a baby from conception). It is also the rate of increase involved in explosions (each bit of explosive sets off more than itself) and atom bombs. It is also another kind of acceleration. Here the acceleration does more than increase. The rate of increasing goes on increasing at the same rate of increasing as the acceleration itself.

## **Pupils' thinking**

This activity is essentially about metacognition and bridging, rather than construction. There is some construction in it for them – that is, in spelling out how the exponential contrasts with a quadratic – but the main agenda is the metacognitive reflection on the qualitative differences between constant speed and these two

different kinds of acceleration, and where these might apply in life. They will not be able to handle the mathematics of it properly without at least a further year of algebra, but the very process of thinking about the comparison as comparing two or three different kinds of possible models provides another push towards, or into, formal thinking.



# 27

## Accelerating the acceleration

### Notesheet 1

#### Steady rise and acceleration

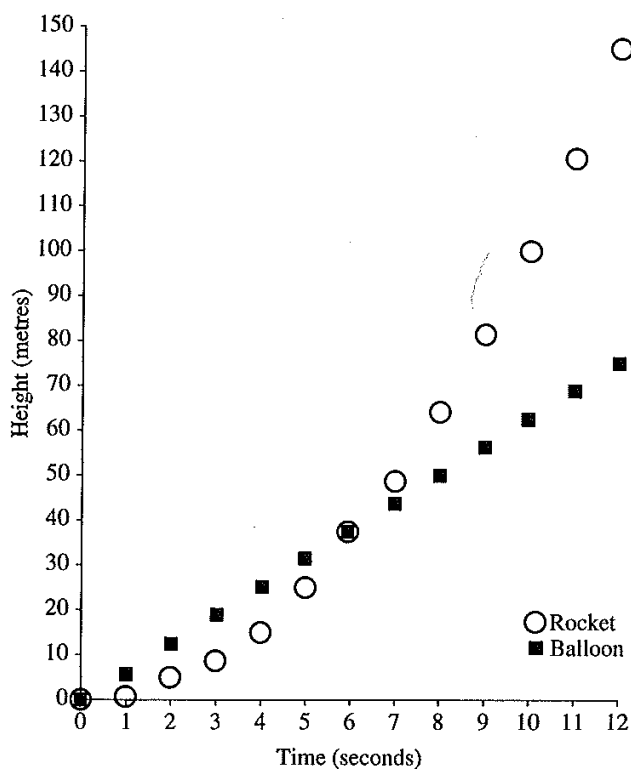
Weather balloon height

Time (seconds)	Height (metre)	Change in height
0	0	—
1	6	6
2	12	6
3	18	6
4	24	6
5	30	6
6	36	6

Rocket height

Time (seconds)	Height (metre)	Change in height
0	0	—
1	1	1
2	4	3
3	9	5
4	16	7
5	25	9
6	36	11

The values from the two tables are plotted on the graph along with new values up to 12 seconds.



How would you describe the **difference** in the way the balloon and the rocket rise? Think and discuss this with your partners, using words like speed and acceleration before sharing with the class.

# 27

## Accelerating the acceleration

### Notesheet 2A

Use a calculator. Use approximate answers using the first two digits followed by zeros.

- 1 Fill in the **middle column** in the table in Notesheet 2B by doubling each time until the weight of the grain reaches one tonne (2 000 000 grains).
- 2 The King's grain store contained 1000 tonnes. The astronomer calculated that by the 30th square, the whole of the King's grain would have been needed!  
By continuing the table, or by another method, find out how he did this calculation, and explain why his calculation was right.

### What accelerating an acceleration means

- 3 Fill in the fourth and fifth columns down to about square 8. Look at the pattern of increases.  
Discuss with your partners how you can explain in words the difference between this and the pattern of increases for the rocket on Notesheet 1.

# 27

## Accelerating the acceleration

### Notesheet 2B

The grain mountain grows!

Square	Number of doubling (2×)	Number of grains	Increase	Increase of the increase
1	0	1	—	—
2	1	2	1	—
3	2	4	2	
4	3	8	4	2
5	4	16		
6	5 (2 × 2 × 2 × 2 × 2)	32		
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				
21				
22				
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27				
28				
29				
30				
31				
32				
33				
34				

An activity on selecting modes of data representation appropriate to different contexts. All the contexts call for comparisons of data sets, but the meanings are best revealed in different ways: either a direct visual contrast, graphs of functions, or correlation scattergrams.

#### Thinking strands

Metacognition and bridging

#### Curriculum links

Handling data: correlation, functions, modes of representation

Organising and analysing data

#### Resources

Notesheets 1, 2 and 3 to be given out together

Extension (Notesheet 4)

Squared and graph paper

### Lesson summary

#### 1 Introduction and small group work (15 min)

Introduce the activity as exploring different ways of analysing three sets of data. Emphasise that one of the most important things a mathematician does is to decide what the question is. Once the question is clarified good ways of finding answers can be explored.

Ask what the pupils understand by 'handling' and 'analysis', and help them to realise in their own way that the task is to group, classify, simplify or represent data in ways that allow some comparisons or ideas to come out. Get them to list data handling methods such as bar-charts, back-to-back stem and leaf, tallying, simple graphs, scatter graphs, mean/median/mode, range.

Draw pupils' attention to three sets of data, going over the structure of each Notesheet. Ask them to discuss in groups how they would approach the data. They should have about 10 minutes before they report back. While they work on the task help some groups to clarify their ideas, without fully doing the 'busy work', and get ready for the discussion.

#### 2 Class discussion (10 min)

Share the different groups' ideas for each of the three data sets, which will mainly be clarifying the main questions in the data sets. Record the suggestions on the board. Accept different ideas if such are evident, and allow all the groups then to choose freely their preferred method of analysis. Emphasise that if a group finds a method unsatisfactory that is also a worthwhile finding, and they should try to think why that is so.

#### 3 Small group work (20 min)

Suggest that different groups start with different data sets, but each should finish at least two. It may be an advantage if you yourself have no prior preference for method, so you are prepared to be persuaded by more

than one method. Even if the groups choose the same method for a data set, the details are likely to be different.

Encourage groups to present their solutions on large enough paper for display during subsequent discussion.

#### 4 Class discussion (15 min)

For each data set separately get groups to present their solutions. They can then summarise which method worked best and why other methods were not helpful. Some of the ideas expressed briefly in the first discussion will be visited again, now with greater insights from pupils. Encourage comparisons between the two sets of data, to emphasise aspects such as:

- In the GCSE results all data are individual: there are two unrelated separate groups to be compared by grades. So each grade (or top two grades, or bottom two grades, and so on) can be a base for comparison, as well as the overall picture of a bar-chart or tally chart.
- In the science experiment there are two types of relationships: the relationship between the same temperatures and values for time, and the similarity between the two tables due to the fact that the entries in one are about twice the other. This allows graphical representation, which may be helpful to estimate times for middle value temperatures and to show the pattern. But bar charts are also feasible, even though they do not allow intermediate results.
- The English/Science grades can be explored along the lines of the correlation lessons pupils have covered, leading to scatter graphs. On the other hand some representative values such as mean, median or mode may be usefully explored too, because of the ordinal numeric sets of ordered pairs, (E, S), for the two groups.

## Mathematical content

This is focused on the Data-Handling aspect of the National Curriculum, but does stray also into the use of mathematical modelling in Science. The first problem is one in which, although the average grade of the girls is higher than for boys, just using averages does not tell the whole story. In fact for Grades A to C (in the results given) there is not much to choose between the boys and the girls' results, whereas the boys are worse in the grades below C. Thus they need a form of representation which will reveal the detail. The ideal way would be two stem and leaf diagrams set back to back, as used in *TM5: Length of words*. But a bar-chart of Grades A to u with the girls' and boys' numbers put alongside each other will do. But first they need to tally the data so as to count the numbers getting each grade.

The second problem actually has two independent variables in it (amount of chemical A, and temperature), and here pupils need to see that the best way of comparing the results is

to use Cartesian representation. In fact the relationship with temperature is an exponential one, and they will see two parallel curves, with one going twice as fast as the other at all temperatures. So the relation with amount of chemical A is a linear one. They may also spot that in both cases, for every 10 degree temperature rise, the speed approximately doubles.

The third problem is set up as a correlation one. The pupils need to 'bridge' to *TM22: Comparing correlations* and realise that if they want to check how well pupils' KS3 English results predict their GCSE results, then what they need to do is to compare the boys' and girls' scatters, again with Cartesian representation. But they would not now be looking for a functional relationship, as in the second problem, but comparing the two correlational ones. When they do this they will see that the assertion is true – however they choose to look at it, the spread of boys' results is wider than that of the girls.

## Pupils' thinking

The whole of this activity is set up to promote metacognition and bridging. At the outset they need to see that they have been given three different contexts, and each probably needs a mode of data representation which will reveal the meaning contained within the data. So here they have to step back from the immediate problems they are given, and search their memory for models of data representation which would best fit each.

Thus the whole class will benefit from hearing the possible solutions and why they were chosen, from each working group. The groups have then to make a choice, and process each of the three data-sets.

Further metacognition should result when they compare and contrast each other's solutions in whole class discussion at the end.

# 30 How do I handle the data?

## Notesheet 1

Here are three investigations to be carried out and compared in the same lesson.  
 You need to handle the data to show the meaning of each set.  
 First discuss each investigation and **how best to deal with it** without doing the work fully.  
 After you have looked at all three investigations, carry your plans out.

### 1 GCSE English results for boys and girls

Newspapers say that girls are doing better than boys at GCSE in English.  
 Maybe this is true. Or maybe there is more to it than that.  
 Here is a part of a school's results, with the pupils' names omitted.

Sex	Grade	Sex	Grade	Sex	Grade	Sex	Grade
G	A	G	G	B	D	B	B
G	C	G	C	B	F	B	D
G	D	G	E	B	A	B	D
G	D	G	C	B	F	B	F
G	B	G	G	B	F	B	B
G	E	G	G	B	F	B	u
G	A	G	E	B	B	B	G
G	C	G	D	B	B	B	G
G	D	G	F	B	C	B	C
G	C	G	F	B	C	B	E
G	C	G	B	B	D	B	E
G	D	G	F	B	B	B	E
G	D	G	D	B	C	B	E
G	D	G	E	B	E	B	F
G	B	G	E	B	E	B	F
G	D	G	E	B	D	B	D
G	C	G	F	B	C	B	G
G	F	G	D	B	C	B	G
G	u	G	A	B	C	B	u
G	D	G	D	B	D	B	A
G	B	G	B	B	E	B	F
G	E	G	C	B	E	B	A
G	D			B	E		
G	C			B	B		
G	D			B	D		

How can you handle this data?  
 Think of possible ways then decide on one.  
 Remember that you are checking what the newspapers are saying.

# How do I handle the data?

## 2 A difficult day in the Science lab

Your science teacher is demonstrating a chemical reaction.

The reaction is between two chemicals and at different temperatures. When the two chemicals are mixed the liquid turns black.

The reaction is complete when the mixture suddenly turns clear and colourless.

You record the time the reaction takes, in seconds.

The teacher first mixes the same amounts of the two chemicals.

Here is the data for different temperatures:

Temperature (°C)	20	30	40	50	60	70
Time in seconds	230	115	58	28	14.5	7

The teacher then **doubles** the amount of the first chemical, keeping the amount of the second the same.

Here is the data for this experiment:

Temperature (°C)	20	30	40	50	60	70
Time in seconds	116	59	29	15	6.5	3.5

How should you present the data for both experiments?

Remember you want to show as much as possible of the meaning of the data, so you need to think about both the temperature and the change in amount of the first chemical.

# How do I handle the data?

## 3 "How good you are at English is a good predictor of how good you are in other subjects." True or false?

Someone says: "You can predict how well **girls** will do in Science just by looking at how well they did in English." Check how true this statement is, then find how well it works for **boys**.

Look at the table below of levels awarded to two groups of pupils at KS3.  
First think of possible ways of handling this, in discussion with your partners.

Girls	English level	Science level	Boys	English level	Science level
	3	1		3	1
	3	3		3	4
	4	2		3	5
	4	3		4	1
	4	5		4	2
	5	2		4	4
	5	4		4	4
	5	4		4	6
	5	4		4	7
	5	5		5	2
	5	5		5	2
	5	7		5	3
	6	3		5	5
	6	4		5	7
	6	4		5	7
	6	5		5	8
	6	5		5	8
	6	5		6	2
	6	7		6	4
	6	7		6	7
	6	8		6	8
	7	7		7	5
	7	8		7	6
	8	8		7	8



# How do I handle the data?

## Extension

Choose one of these two problems. Decide what data you would collect and how you would handle the data in order to try to solve the problem.

- 1 Someone claims that people's height and hand-spans go together.  
What data would you collect to check this, and how would you process it?
- 2 You want to choose between two holiday resorts, Ilfrapool and Blackcombe.  
You want to be sure of getting a lot of sun for the last week in July. You find records for each town of the number of hours of sun that week in each of the resorts over the last twenty years. At first glance you can see that Blackcombe varies between 35 and 53 hours, whereas Ilfrapool can be as low as 23 and as high as 70 hours.

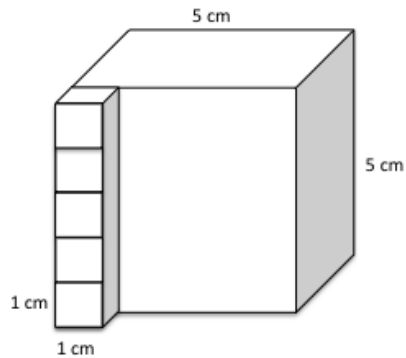
How would you use the records to decide which is the best bet?

How would you make the comparison?

Can you show two possible patterns there might be in the data which would lead to different decisions?

### C. Pilot Study: Reasoning Skills Test

1.



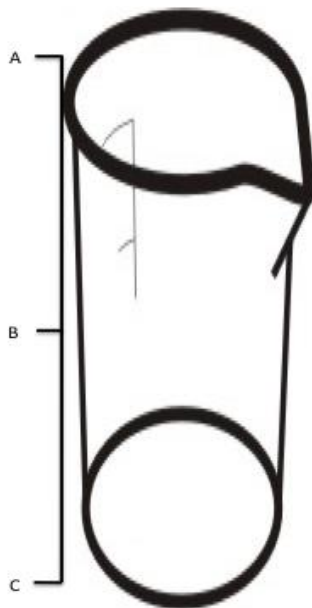
This plasticine block is formed only by cubes.  
¿What is the volume of the block?

Answer .....

2. ¿How much water would spill over if we submerge the previous block under a container full of 2 litres of water?

Answer .....

3.



If we submerge the block under this jar full of water and we maintain it until level A, some water is spilled over. ¿What happen if we now submerge the block to level B? ¿How much water would spill over in comparison when we did it to level A?

- a) less water
- b) same water
- c) more water
- d) not possible to determine

¿And if we submerge it to level C?

- a) less water
- b) same water
- c) more water
- d) not possible to determine

4. If the block is converted into a ball and we submerge the ball under the water.

¿How much water would spill over in comparison when we submerged the block?

- a) less water
- b) same water
- c) more water
- d) not possible to determine

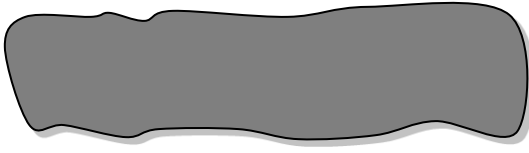


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5. And if the ball is now converted into a cylinder.

¿How much water would spill over in comparison when we submerged the ball?

- a) less water
- b) same water
- c) more water
- d) not possible to determine



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6. And if we use a metallic block of the same size as the plasticine block and we submerge it completely under the water. ¿How much water would spill over in comparison when we submerged the plasticine block?

- a) less water
- b) same water
- c) more water
- d) not possible to determine

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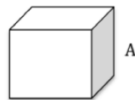
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¿Why?

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7.



Blocks A and B are made of the same metal. A weights 60 grams and its volume is 15 cm<sup>3</sup>. B weights 160 grams. ¿What is its volume?  
\_\_\_\_\_ cm<sup>3</sup>.



¿How did you calculate your answer?

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8.

Box A is full of alcohol and it weights 1,500 grs. Box B (which is twice taller than A) is full of water and it weights 2,000 grs, If you put box A over box B:

- a) Would float
- b) Would sink
- c) It is not possible to determine

How did you find your answer?

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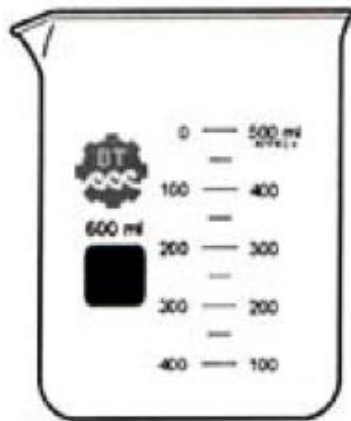
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9.



a. How did the king calculate the volume of his new and old crown using a glass beaker?

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b. Then the king weighted both crowns and he realized that the new crown (which is bigger than the old one) was heavier than the old one. Based on that, he concluded that the new crown is made of a lighter material than the old one. ¿How did he draw that conclusion?

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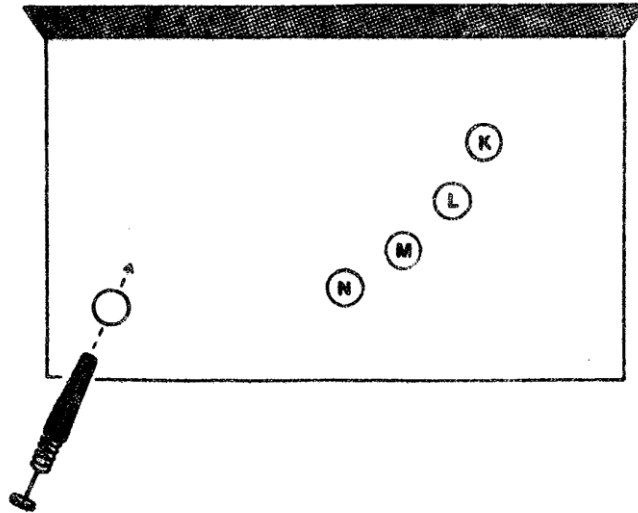
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10.

A rubber ball is rolled across a smooth flat table to hit a perpendicular wall. Which of the other balls will it hit?

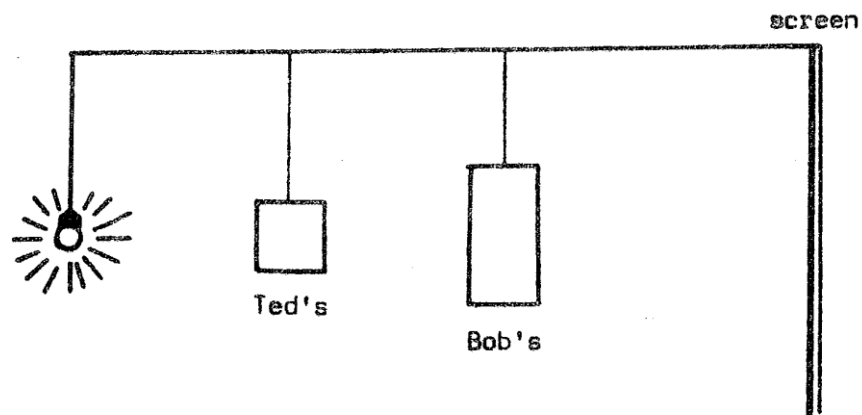


- A. Ball K.
- B. Ball L.
- C. Ball M.
- D. Ball N.
- E. Any of the balls.

(2/LOT01)

11.

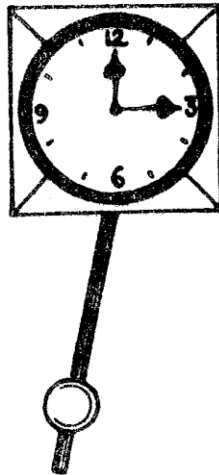
Ted and Bob were hanging two tins in front of a lamp as shown in the picture. Bob's tin was twice as tall as Ted's. Which of their tins produced the largest shadow on the screen?



- A. Ted's tin since it is further away from the screen.
- B. Both produce the same shadow since the ratio between size of the tin and distance from the lamp is the same.
- C. Bob's tin since it is biggest.
- D. Ted's tin since the greater the distance from the screen, the larger the shadow.
- E. Bob's tin since the shadow depends on the size of the tin.

12.

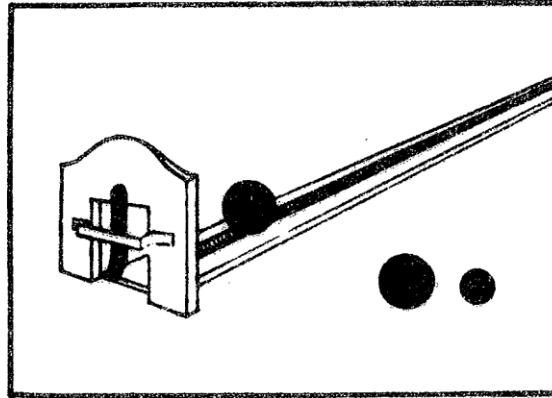
Bob had made a clock as shown in the picture. In order to make the clock go he had attached a pendulum having a weight of 200 grams. But the clock was too slow. What shall he do to speed up the clock?



- A. Give the pendulum a stronger push to start it.
- B. Let the pendulum start higher up and use a heavier weight.
- C. Exchange the weight for a lighter weight placed further down on the pendulum.
- D. Exchange the weight for a lighter weight placed higher up on the pendulum.
- E. Move the weight higher up on the pendulum.

13.

Jeremy is pushing two balls, one after the other, along a horizontal plane with a spring as shown in the picture. One of the balls is bigger and heavier than the other. Which ball will go furthest?



- A. The small one since it is easier to set in motion.
- B. The big one since it is heavy.
- C. The small one since it has less friction and air resistance.
- D. The big one since the small one is lighter.
- E. The small one since the big one stops earlier.

14.

During an archaeological excavation, four different types of knives were found as shown in the picture. The colour of the blades and handles of the knives was light or dark. Was there any relationship between the colour of the blades and handles?

3 knives

1 knife

2 knives

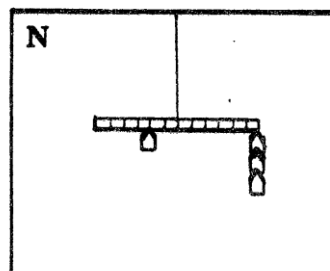
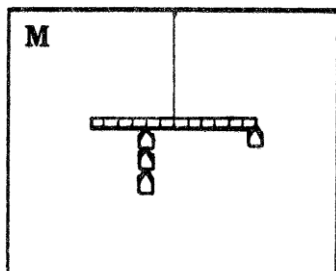
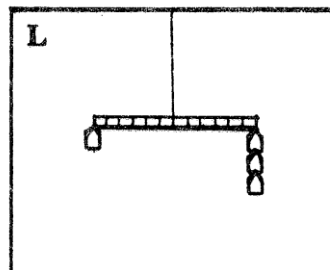
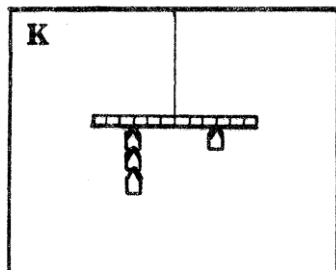
6 knives



- A. 9 knives had the same colour on both blades and handles whereas 3 had a different colour.
- B. There were more dark knives than light knives.
- C. 6 knives had dark blades and dark handles whereas only 2 had dark blades and light handles.
- D. There was no relationship because there was a different number of knives of each type.
- E. Out of 4 knives having light blades, 3 had light handles.

15.

Anne was hanging some paper-clips on her ruler. Which picture shows the best way for Anne to hang her paper-clips in order to balance the ruler?





- A. Picture K.
- B. Picture L.
- C. Picture M.
- D. Picture N.
- E. Any of the pictures.

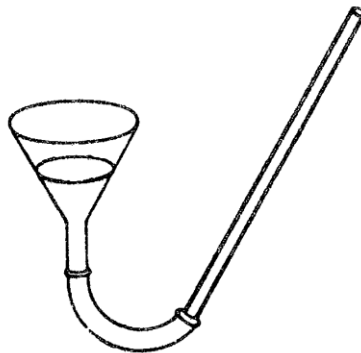
16.

Kate had a big wooden ball and an ordinary little door-key made of metal. She wanted to see if they could float on water. The ball was heavier than the key. Yet the ball floated while the key sank to the bottom. Why did the little key sink?

- A. The key is made of metal and metal always sinks.
- B. The key can't float on the water because it is too light.
- C. The key sank because the water weighed down the key.
- D. The key is too heavy for the water so the water can't carry it.
- E. The key is heavier than the same volume of water.

17.

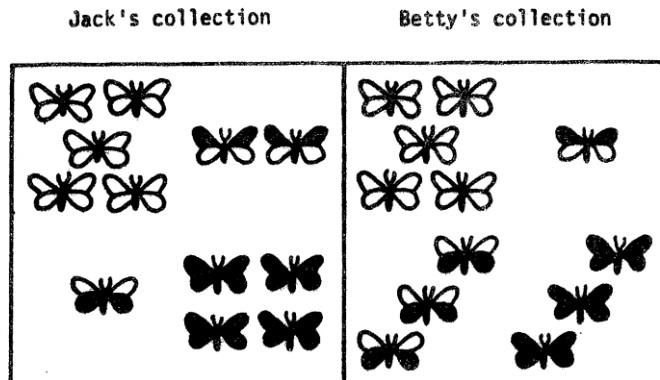
Margaret connected a funnel and a thin glass tube using a rubber tube. She poured water into the funnel up to the line on the funnel as shown in the picture. What is the best explanation of to what level the water will rise in the thin tube?



- A. As the pressures of the columns of water balance each other the water will go as high in the tube as in the funnel.
- B. The water will go nearly as high in the tube as in the funnel because it goes through the pipe and into the tube.
- C. Margaret tilts the tube and therefore the water doesn't go as high in the tube as in the funnel.
- D. As the tube is thinner than the funnel the water will go up higher in the tube than in the funnel.
- E. The water will go as high in the tube as in the funnel.

18.

Jack and Betty had gathered butterflies. Which of their collections shows the strongest relationship between the colour of the backwings and the colour of the frontwings?



- A. The same in both collections because they have 5 butterflies with white frontwings as well as white backwings.
- B. Betty's collection, because 5 out of the 6 butterflies, which have white backwings, also have white frontwings.
- C. Jack's collection where 9 butterflies have frontwings and backwings of the same colour as compared to 3 of a different colour.
- D. The same in both collections because there are the same number of butterflies in both of them.
- E. Jack's collection which has 4 butterflies with dark frontwings and dark backwings but only 1 with white frontwings and dark backwings.

19.

Mary and Jane each bought the same kind of rubber ball. Mary said, "My ball bounces better than yours." Jane replied, "I'd like to see you prove that." What should Mary do?

- A. Drop both balls from the same height and notice which bounces higher.
- B. Throw both balls against a wall and see how far each ball bounces off the wall.
- C. Drop the two balls from different heights and notice which bounces higher.
- D. Throw the balls down against the floor and see how high they bounce.
- E. Feel the balls by hand to find which is the harder.

20.

Ted was constructing a model of a submarine. He wanted to see if a closed tin could stay half way between the surface of the water and the bottom, not floating nor sinking to the bottom. He poured some water into the tin - and look! He succeeded! Why?

- A. The tin can't float on the top of the water because it is too light.
- B. The tin with some water inside is as heavy as the same volume of water.
- C. The tin is not too heavy and there is enough water to carry it.
- D. The tin is not completely filled with water.
- E. The water is not strong enough to push the tin to the bottom.

## D. Pilot Study: Field Notes Summary

### Session 1- Activity 2: Tournaments

In this activity, students have to solve the following problem: Teams from eleven schools are playing in a sports tournament, where each team plays every other team at home. How many matches are played in total?

At the beginning of the session, students did not manage to understand the problem deeply, specially the instruction that said that every team has to play against every other team at home. For this reason, some students answered promptly numbers such as 11, 69 or 80 matches. However, when all the class correctly understood the problem, they used three different methods to solve it: (i) enumeration, (ii) lines and arrows, and (iii) multiplication. After this step, most students easily comprehended that each school plays one match less than the total number of schools participating in the tournament (because each school plays against every other school except itself). In consequence, students developed the formula  $x(x - 1)$  to calculate the number of matches played in a tournament with  $x$  number of schools, where 'x' represents the number of schools participating in the tournament and ' $x - 1$ ' the number of matches that every school plays. Getting to this formula was not a big challenge for students, thus they did not develop long discussions about the problem.

### Session 2- Activity 15: Circle Functions

This activity presents the next problem to the students: The sport department of your schools is planning to build a football field that should be half size the original one. According to the football teacher, if you diminish the radio length of the central circle to its halve, it will be 50% smaller than the initial one. Do you agree with him?

At first glance, students seem to be motivated with the activity because they engaged with it and were putting their effort in trying to solve it. When I asked them preliminary, if they agreed with the football teacher, most of them replied yes. In other words, the majority thought that by diminishing the radio to its half the area would be 50% less than the original one.

Apparently, they experienced some difficulties in thinking about a problem in abstract terms, since most of them replaced the variables (i.e. radio) for real numbers, in order to check if their belief was right. Therefore, instead of thinking about the relationship between a radio  $r$  and its area  $A$  ( $\pi r^2$ ), they chose a case where the radio was 6 for example ( $A = 113.04$ ) and compared the area with when the radio was 3 ( $A = 28.26$ ). Using examples, they finally concluded that a circle has an area 50% smaller than an other circle, when the original radio diminishes to its 75%. Although the statement is correct, they only managed to get to it using examples, without understanding the relationship between a radio and its area.

Having done that, I asked them if they can think of a way of estimating (without much calculation) the area of a circle. Most of them suggested that it was possible by calculating the square of the radio (i.e. if the radio was 8 they estimated the area as  $64\pi$ ) and only a small group of them said by rounding off  $\pi$  to 3 (i.e. if the radio was 8 they estimated the area as 192).

Apparently, they knew the formula for calculating a circle's area but they faced some problems in understanding the stated relationship. In addition, I observed difficulties in performing the inverse operation, in other words, they found easier to calculate the area if they know the radio than calculating the radio based on the area.

After that, students had to represent in a graph how perimeter and area were changing while the radio was increasing. Here I observed the peak difficulty of the class, because only one student was able to explain why the perimeters form a straight line and the areas a curve, by saying that the last one is an exponential function.

To sum up, I would argue that this activity was more adequate in terms of complexity, but it was still difficult to generate cognitive conflict, since students mostly applied previously memorized maths contents, without leaving space for generating unexpected situations.

Another problem during this activity was that, for the first time, I realized that for conducting a CA activity and for fulfilling all its potential, it's necessary to have mathematical knowledge, not only about the content but also about the pedagogy of it. I don't have the necessary mathematical knowledge to explain it clearly to the students because I'm not a teacher but an educational psychologist. I felt that although I understood the activity and the way it is supposed to be solved, I lack the deep expertise to produce the expected effect on students.

### Session 3- Activity 20: Heads or Tails?

During activity 20 students are supposed to develop the subsequent problem: Next week your school will be running a football championship and each class has to present two different teams. Your students have decided they prefer to be randomly allocated into one of them. In consequence, you proposed that each of them throws a coin. One team will be the "heads" one and the other the "tails". However, some students started to complain since they think that if the person before them gets a 'head' they will probably get a 'tail'. So they do not have the same possibility of getting a head than a tails. Do you agree with them?

When asked them what they did think about their students statement, all of them agreed that when throwing a coin you have 50% of chances of getting a head and 50% of getting a tail. In addition, all of them stated that what you get when you throw a coin the first time, is not related with what you get when you throw it the second time. For exploring if their ideas were correct, they started to develop the experiment of throwing a coin 50 times in a row. The part of the Notesheet that seemed to be the most difficult for them, was the questions that said 'What should you do for comparing the number of heads you got in samples with different samples sizes?'. This was the first time I realized they had difficulties for understanding and applying proportions and percentages.

For the second activity, students needed to find the pattern which described how many times they got  $x$  (1, 2, 4, 5...9) number of heads in a row (i.e. H, HH, HHH, etc.). When they summed up how many times that happened to the whole class, it was 36 for 1 head in a row and 26 for 2 heads in row. However, they were not

able to suggest any pattern to describe it. For that reason, I asked them: “How many more times you got 1 head than 2 heads?”. And few of them replied: “10 times more”. So again, I concluded they have problems in comprehending the difference between ‘how many times more’ and ‘how much’ which is exactly the same difficulty I noticed about proportions in the first part of the session.

In this final part, they had to find an explanation for the fact that while the number of heads in a row increased, the number of time the class got it decreased. As they were not able to do so, I tried to explained them the fact that for calculating the chances of getting a successful event (‘head’) many times in a row (2, 3, 4, 5...9), you have to multiply the chances of getting 1 successful event by the times you want to estimate. For example, if the chances of getting one ‘head’ is  $\frac{1}{2}$ , the chances of getting 3 ‘heads’ in a row is  $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$ . So the chances decreases exponentially.

The point of describing in details the previous maths explanation, is giving an example of the problem I introduced at the end of session 2 field notes. As I said there, I do not have a mathematical background, so when I have to explain these maths contents to the students, I have the impression that the session does not finish its course in the way it was supposed because I do not have the necessary content and pedagogical skills.

#### Session 4 and 6- Activity 22: Comparing correlations?

This activity presents three different graphs that represent the relationship between the amount of use of three mathematical software and students’ achievement in three different tests. Based on them, students are encourage to find out which of them have the strongest relationship between the two variables so they can make a decision about what software should buy their schools.

I found out that there are two elements that are the most difficult for the students: (i) use and achievement variables are expressed in three different ways in the three graphs. For example, mathematical performance is measured by grades average obtained in mathematics (graph I), % of right answers in the GCSE (graph II), and % of right answers in the final exam (graph III). This produced high amount of complexity, because students focused their attention in evaluating or judging which was the most valid way to measure mathematics achievement, rather than analyzing which case demonstrated to have the strongest correlation between software use and mathematic performance. They were not able to think only about the relationship expressed in each graph without considering the type of variables represented in them.

The second difficult element (ii) for them, was that the graph III had much more cases than graphs I and II. In consequence, they argued that it was not possible to compare graphs with different number of cases. This fact reinforces again the idea that they experiences difficulties in thinking about proportion and percentages, which is exactly the way of comparing samples with different numbers.

It was very difficult for then to find out a way of analysing the relationship of each graph. Some students suggested to count the number of high achievement cases in each graph to evaluate which software is the best for improving students’ performance. However, they did not realize that the activity’s purpose was to

relate high achievement with high use and not high achievement itself. The only graph that was relatively easy to analyze, was the number II since its round shape suggested that there was not any relation between use and performance. In contrast with graphs I and II that formed a line.

It is relevant to observe that the course of this activity was smoother than previous ones and I also felt very comfortable and confident during it. When I analyzed this fact, I realized that it was because the skills involved in the activity were more general and transferable to other contexts. For this reason, there was no necessity to explain any mathematical content or even to talk using maths terms. The activity was focused on trying to understand three different graphs within a professional framework, so we did not have to use any concept like average, mode or median (maths terms which are generally used to talk about correlation), to find out the strongest relationship between use and achievement. We did not even use the word correlation to name what we were trying to do.

Based on that, I realized that I was wrong when I concluded that I am not able or I do not have the necessary expertise to conduct cognitive acceleration in mathematics sessions. What I need to do is to choose the adequate activities or adapt them in the proper way, in order to have activities that do not involve the usage, at least explicitly, of math contents or terms. Even more, as I explained previously in this chapter, choosing mathematics as the subject for my cognitive acceleration programme is merely a practical decision. Therefore, math is not related with the aims of the programme since its main objective is to foster thinking skills, general ones, that can be used in any future and professional context. So at the end, mathematics are only a tool for accomplishing that purpose.

#### Session 5 and 8- Activity 30: How do I handle the data?

The purpose of this activity is to try out the best way of organizing three sets of data in order to understand what is going on. It was very interesting to find out that, although most students managed to represent the data properly (i.e. table, graph, etc.), it was difficult for them to understand the meaning of it and to draw conclusions.

It is important to remark that, for the three data sets, most students used tables to represent the data rather than graphs, even in some cases where graphs would have been more helpful and enlightening. In both sessions, only one or two students created graphs and during the discussion, some classmates argued that this is not a correct way to represent that data because we had two different groups (i.e. women and men), so we would not be able to use only one graph to represent both of them. Based on this, it is possible to conclude that they might have difficulties in representing and comprehending graphical data.

Here I observed again their problem for using proportionality, since when I asked them which was the pattern behind the relationship between temperature and how much a chemical reaction lasts, they were not able to describe it (each time the temperature was raised by  $10^{\circ}\text{C}$ , the reaction time diminished to its 50%).

Finally, in both sessions nobody was able to organize the data of the third data set which presented students' math and science grades and they had to explore if there was a relationship between the two performances.

As a conclusion, I would suggest that this session was very useful for generating cognitive challenge in the students since it was difficult enough and they engaged with the activities proposed. However, my impression was that it was not possible to analyse three data sets in only one session that last 50-60 minutes approximately because it did not leave the necessary space to discuss and to develop each data set in depth.



## E. Main study: Interview Protocols

### E.1. Interview protocol: Before the CAME course

#### Warm-up

Thanks for taking part today. I wonder if you could start by telling me a little bit about yourself and why you have decided to make teaching your career choice.

#### Views about teaching, learning and thinking

1. From what you have learned from university and your experience in the classroom about the way that children learn, ¿How would you describe the way that children *learn*?
  - 1.1. ¿Are there any particularities about *learning maths*?
2. ¿How would you describe the process of *teaching* and the role that teachers have in helping children learn?
  - 2.1. ¿Are there any particularities to *teaching maths*?
3. ¿What role do you see *thinking* plays in learning?
  - 3.1. ¿And in *learning maths*?
4. ¿Have you been surprised of anything that you have noticed about learning?
  - 4.1. ¿And of learning maths?
5. ¿What do you understand by thinking?
6. ¿What would you expect from a course that it is called 'Thinking development'?
7. The activities you're going to work on encourage children to think in mathematics. ¿How important do you think this approach would be in the classroom?
  - 7.1. ¿Why?

#### Attitudes towards teaching and learning

8. ¿How do you feel about teaching?
  - 8.1. ¿And about *teaching maths*?
9. ¿What are your *strengths* and the areas you need to work on as a prospective teacher?
10. ¿How do you feel about promoting the development of thinking skills in your students?
11. Some research suggests that some children really hate maths and so avoid doing it whenever they can. Have you experienced this in the classroom and why do you think this happens?
  - 11.1. And do you think this phobia can be overcome or reversed?
12. Can you tell me more..?
13. How does that link with what you said earlier...?
14. Are there any implications from that for teachers or learners?"

## **E.2. Interview protocol: After the CAME course**

### Introduction

Thanks for taking part today.

### Perceptions about the CAME course from a student point of view

1. Do you think that the course has helped you with your ability to think and to understand mathematics?
  - a. How?
2. How would you describe the course?
  - a. What was similar and different to what you were expecting from it?
3. How would you describe the activities we used during the course?
  - a. Which aspect was the most interesting for you?
  - b. Why?
4. Do you think that the course was similar to other mathematics courses you have had during your experience as student?
  - a. Why?
5. What do you think you learned in terms of comprehending mathematics?
6. Do you think that the course was useful in terms of your learning/professional development?
  - a. How?
7. Do you think that the course had any impact on your ability to think/reason?
  - a. How?
  - b. Which?
8. Do you think that the course had any impact on your confidence about teaching mathematics?
  - a. How?
  - b. Which?

### Perceptions about the CAME course from a prospective teacher point of view

9. Do you think that the activities are useful for teaching primary mathematics?
10. Do you think that the activities are useful for learning primary mathematics?
11. Do you think that you would be able to apply what you learned in your own classroom?
12. What would you say about the importance of teaching thinking skills?
13. How confident you feel about teaching thinking skills in your own classroom?

## F. Main Study: Participants Information Sheet

### F.1. Participants Information Sheet: Experimental Group

#### INFORMATION SHEET FOR PARTICIPANTS

REC Reference Number: REP(EM)/10/11-44

YOU WILL BE GIVEN A COPY OF THIS INFORMATION SHEET



#### “Developing and Evaluating Formal Thinking Skills in Prospective Primary Teachers”

We would like to invite you to participate in this postgraduate research project. You should only participate if you want to; choosing not to take part will not disadvantage you in any way. Before you decide whether you want to take part, it is important for you to understand why the research is being done and what your participation will involve. Please take time to read the following information carefully and discuss it with others if you wish. Ask us if there is anything that is not clear or if you would like more information.

The research project's aim is to assess the impact of a cognitive acceleration program in prospective primary teachers' thinking skills in Chile. For this purpose, we will recruit about 15 students who are in their fourth or fifth year of their Bachelor in Education degree at your University. In other words, any student who is studying a Bachelor in Education degree but is in other year different from the fourth or the fifth one can not participate in this study. If you decide to take part in our study, your participation would consist in:

- (i) Answering a multiple choice and open ended question test at the beginning and at the end of the course.
- (ii) Writing learning journals every other session of the course
- (iii) Attending 18 lessons, which last 50-60 minutes, delivered by a member of our staff once a week
- (iv) Some of you are also going to be invited to participate in an interview at the beginning and at the end of the course that would last no more than 60 minutes

Although your participation is very time-consuming, you could benefit from it since this type of programmes have shown improvements in their participants' academic performance. If during that time you do not feel comfortable about going to the sessions or simply you do not want to participate anymore you can leave the programme at any moment without experiencing any harmful consequences.

To ensure confidentiality, the only two persons who will have access to your data are the two researchers in charge of the project. In this sense, your information will not be disclosed to any university authorities or tutors. In addition, your data is going to be used only once for the purposes described in the consent form. Therefore, if the research team want to use that data again in the future, they would have to ask you for a new consent. If you need further information or have any queries please contact the researcher Bernardita Tornero, [bernardita.tornero@kcl.ac.uk](mailto:bernardita.tornero@kcl.ac.uk).

It is up to you to decide whether to take part or not. If you do decide to take part you will be given this information sheet to keep and be asked to sign a consent form but you are still free to withdraw at any time and without giving a reason. In addition to withdrawing yourself from the study, you may also withdraw any data/information you have already provided up until November 2012 when it is going to be transcribed for use in the final report.

## F.2. Participants Information Sheet: Comparison group

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If you decide to take part in our study you will have to take a paper and pencil test which consists in 22 multiple-choice format questions with four possible answers (a, b, c, d) and lasts about 45 minutes. If you think that the test is too difficult or simply you do not feel comfortable about it you can leave any questions in blank or leave the entire test at any moment without experiencing any harmful consequences. It is worth noting that you will not have access to your or any other participant tests results and to the tests answers either. However, we offer you the possibility of get a copy of the final report which will tell you the performance of the participants as a group. In that case, we will ask for an e-mail address in order to send you a copy about 2 months later.

To ensure anonymity and confidentiality we will not ask you for any other personal information than name. If you do decide to take part you will be given this information sheet to keep and be asked to sign a consent form but you are still free to withdraw at any time and without giving a reason. In addition to withdrawing yourself from the study, you may also withdraw any data/information you have already provided up until November 2012 when it is going to be transcribed for use in the final report. The only two persons who will have access to your data are the two researchers in charge of the project. If you need further information or have any queries please contact the researcher Bernardita Tornero, [bernardita.tornero@kcl.ac.uk](mailto:bernardita.tornero@kcl.ac.uk).

## G. Main Study: Participants Consent Form

### G.1. Participants Consent Form: Experimental Group

CONSENT FORM FOR PARTICIPANTS IN RESEARCH STUDIES



Please complete this form after you have read the Information Sheet and/or listened to an explanation about the research.

"Developing and Evaluating Formal Thinking Skills in Prospective Primary Teachers"

King's College Research Ethics Committee Ref: Education and Management Panel (E&M REP)

Thank you for considering taking part in this research. The person organizing the research must explain the project to you before you agree to take part. If you have any questions arising from the Information Sheet or explanation already given to you, please ask the researcher before you decide whether to join in. You will be given a copy of this Consent Form to keep and refer to at any time.

**Please tick  
or initial**

I understand that if I decide at any time during the research that I no longer wish to participate in this project, I can notify the researchers involved and withdraw from it immediately without giving any reason. Furthermore, I understand that I will be able to withdraw my data up to the point of analysis (November 2012).

☐

I consent to the processing of my personal information for the purposes explained to me.

☐

The information you have submitted will be published as a report and you will be sent a copy. Please note that confidentiality and anonymity will be maintained and it will not be possible to identify you from any publications.

☐

Participant's Statement:

I -

\_\_\_\_\_  
I agree that the research project named above has been explained to me to my satisfaction and I agree to take part in the study. I have read both the notes written above and the Information Sheet about the project, and understand what the research study involves.

Signed

Date

Investigator's Statement:

I \_\_\_\_\_

Confirm that I have carefully explained the nature, demands and any foreseeable risks (where applicable) of the proposed research to the participant.

Signed

Date

## G.2. Participants Consent Form: Comparison Group

### CONSENT FORM FOR PARTICIPANTS IN RESEARCH STUDIES



Please complete this form after you have read the Information Sheet and/or listened to an explanation about the research.

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King's College Research Ethics Committee Ref: Education and Management Panel (E&M REP)

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Please tick  
or initial

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☐

Participant's Statement:

I -

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—  
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Signed

Date

Investigator's Statement:

I \_\_\_\_\_

Confirm that I have carefully explained the nature, demands and any foreseeable risks (where applicable) of the proposed research to the participant.

Signed

Date

## H. Main Study: Interviews information sheet

### INFORMATION SHEET FOR PARTICIPANTS

REC Reference Number: REP(EM)/10/11-44

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If you decide to take part in our study, your participation would consist in attending 18 lessons, which last 50-60 minutes, delivered by a member of our staff once a week and answering two interviews: one in August and the last one in December. Although your participation is very time-consuming, you could benefit from it since this type of programmes have shown improvements in their participants' academic performance. If during that time you do not feel comfortable about going to the sessions or simply you do not want to participate anymore you can leave the programme at any moment without experiencing any harmful consequences.

To ensure confidentiality, the only two persons who will have access to your data are the two researchers in charge of the project. In this sense, your information will not be disclosed to any university authorities or tutors. If you need further information or have any queries please contact the researcher Bernardita Tornero, [bernardita.tornero@kcl.ac.uk](mailto:bernardita.tornero@kcl.ac.uk).

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## I. Main Study: Interviews consent form

### CONSENT FORM FOR PARTICIPANTS IN RESEARCH STUDIES



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#### "Developing and Evaluating Formal Thinking Skills in Prospective Primary Teachers"

King's College Research Ethics Committee Ref: Education and Management Panel (E&M REP)

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Please tick  
or initial

I understand that if I decide at any time during the research that I no longer wish to participate in this project, I can notify the researchers involved and withdraw from it immediately without giving any reason. Furthermore, I understand that I will be able to withdraw my data up to the point of analysis (November 2012).

☐

I consent to the processing of my personal information for the purposes explained to me.

☐

The information you have submitted will be published as a report and you will be sent a copy. Please note that confidentiality and anonymity will be maintained and it will not be possible to identify you from any publications.

☐

Participant's Statement:

I -

—  
agree that the research project named above has been explained to me to my satisfaction and I agree to take part in the study. I have read both the notes written above and the Information Sheet about the project, and understand what the research study involves.

Signed

Date

Investigator's Statement:

I \_\_\_\_\_

Confirm that I have carefully explained the nature, demands and any foreseeable risks (where applicable) of the proposed research to the participant.

Signed

Date

## J. Code List

		CODE NAME	CODE DESCRIPTION	CODE QUOTATION
CAME	EXPERIENCE	CE_CONFIDENCE	This code is used when they claim that participating in the course has improved their confidence to teach math or to share their reasoning with their peers	I do think that now I feel more confident about developing these skills in my students...because ...before I always tried to apply formulas in a very rigid way, and with the course I realized that you're allow to use other abilities, take other routes, use different strategies, to get to a result. (ULA-Macarena M 2)
		CE_MATHSKILLS	This code is used when they claim that participating in the course helped them to realize that they are good at mathematics, they started to like them, they have math skills or they have improved their grades/comprehension in their math subjects; or they have a better vision of what learning/teaching math is; or how to promote maths skills in their students in a better way of how they have been taught	At the end of the course I found out that I do have mathematics skills...because I thought I didn't. I was convinced that I was useless, useless, useless,...at school and at the University I already had failed math. (UCINF-Claudio G 2)
		CE_METACOGNITION	This code is used when they claim that the course helped them to be more aware of their own learning/thinking process	...and at the end of each session you always asked us 'how did you do it?'...and you made us to write the different reasonings skills we had used, in some sense to do the closing, the metacognition...For me that was important, because you can solve the problem and that's it, but thinking about how we did it? that was important...maybe for being aware of what it was missing... (UFT-Francisca M 2)

		CE_MOTIVATION	This code is used when they claim that during CAME lessons they have fun or the activities were motivating; or they could use this type of activities to motivate their students in the future	...apart from that, the most fun classes, if you see, are the activities that promote a kind of discussion inside the class, like 'hey! I think this' and, at the end, everyone has to support what they did...All that makes you keep thinking, sharpening...'hey! I can do this, I can do that'. It keeps you awake all the time...in other words, it has an impact on motivation... (ULA-Antonia B 2)
		CE_THINKINGSKILLS	This code is used when they say that the course has been useful in helping them to have the tools to design and orientate their lessons (today or in the future) to the different thinking skills p.e. identify, compare, contrast, infer, etc.; that they had to use their own reasoning skills to solve the problems; or they have developed their own thinking skills	I think...at the end is like...to practice the way of thinking mathematics in order to be able to transfer it to your students. In that sense, if teachers change the way they think, they will teach in a different way. Therefore, to have a more analytical thinking, which is what the course tried to accomplish, more critical... (ULA-Jacinta V 2)
	IMPROVEMENTS	CI_APPLY	This code is used when they claim that the course could be improved by having a second module where they have the chance to put into practice with their classmates what they have learned	The only thing I'd add to the course is that you give us...for example, the opportunity or the moment for us to design a class for our classmates and see if we're doing it right in the sense of developing thinking skills. (UFT-Trinidad L 2)
		CI_STUDENTS	This code is used when they claim that the course could be improved by having more students participating in the discussion	The course went far beyond my expectations, but I think it'd have been even better if there were more classmates to contribute to the discussion. (UCINF- Claudio G 2)
		CI_TIME	This code is used when they claim that the course could be improved by having longer classes every week or having more classes during the term	...because many times I had the feeling that we were going very fast, we had to solve the problems fast, because many classmates had to leave on time because they had other courses afterwards...so I think it's important that the course would have had two hours and not only one. (ULA- Macarena M 2)

	METHODOLOGY	CM_CONST RUCTIVIST	This code is used when they claim that during CAME lessons they learn through doing, actively, with real life problems or by following a methodology that was coherent with a constructivist view of teaching and/or learning	I think that the teacher's role is also important. The University has the plan, the mission of creating constructivist teachers, but I haven't seen it in all my courses, but this course makes a contribution to this constructivist training that the University is looking for, to create constructivist teachers. (UCINF-Claudio G 2)
		CM_DIVERSITY	This code is used when they claim that the methodology/problems we used during the course emphasized a flexible approach to the problem in the sense of giving the students the chance to solve it in many different ways or from different points of view	Well...I wrote about this on my learning journal, but I think [the activities] were adequate because it's different if you've told us: 'What would I get if I mix red and yellow?', everyone had answered orange or different variations of orange. But the situations that you gave, gave space for all of us to think in different ways, each may have done different kinds of analysis. It helped us to realize that, really, we don't think in the same way, they not everyone made a bar chart, some did dispersion charts or maybe a third different kind...you usually think that everyone will do the same as you. (UFT-Francisca M 2)
		CM_NOVELTY	This code is used when they claim that the methodology we used during the course was new for them and very different from what they are used to in other math courses or at school; or that is new for them to talk about the relevance of developing thinking skills in their students	...they [the Department of Education] never told us how this course was going to be, but when you presented us the first session and the activity, I found it novel and different. (...) it was something we weren't used to it. From the first activity you realize that this class is going in another direction, that is not similar to the other [courses], that you're going to learn something meaningful, that you're going to benefit from this workshop, from this knowledge, from these group experiences. So at the end, you come for that reason, not just for the assistance. (UCINF-Nicole B 2)

		CM_SHARING	This code is used when they claim that having to share was useful to learn from their peers, from their own mistakes or to be conscious that there are different kinds of thinking and leaning styles or it helped them to verbalize their own thinking	You always think that you're right until you listen to your peers. And when you listen to your peers, you realize that, in fact, plan b and c are also good options and many times you hadn't even thought about them. And it's not wrong if you didn't think about them, that's why we're social beings, for that reason whe complement to each other. I feel that was a great contribution to the course. (ULA-Laura A 2)
MATH	LEARNING	ML_ABILITY	Being able to learn maths is related to a general ability in the sense that people that is good at maths is because they are super intelligent or they have certain kind of ability	Wow mathematics! For some reason people say that people who studies Mathematics are super smart, because they're really intelligent, they have to learn a lot, if they forget some...well...if they are solving a very long problem and they forget how to solve a part of it and they get a number wrong, everything is wrong. You see? Because I can keep solving the problem and I'll get to a result but if I made a mistake at the beginning, everything will be wrong. (UCINF-Claudio G 1)
		ML_CONCRETE	This code is used when they refer to learning maths as being concrete in the sense, that it should start with something familiar to their students, previous knowledge or in contact with their own reality or experience	...mathematics are super...not lately, more concrete materials are been used lately, with more drawings and closer to students' reality, but it's super abstract, that's why it becomes much more difficult to teach than to tell a story in language for example, if you know what I mean... (UFT- Valentina R 1)

TEACHING	MT_BASIC	This code is used when they say that the maths they teach in primary school is basic, simple or "easy"	I feel a bit weak maybe. I mean, I have mix feelings, because my intention is to work in first and second grade, obviously everything they [students] see in first and second grade I already saw it. I already was in first and second grade, I had maths until the last year of high school, at university I have studied math during the four years, so it is unlikely that they will ask something you don't know. (UCINF- Karina M 1)
	MT_COMPLEX	This code is used when they say that teaching maths is complex	Yes, the math teacher has a harder job than the others, because teaching other subjects is easier since you can find a way to motivate your students, something that can be interesting for them, familiar. But in the case of math, it's like...at the end there are numbers, so it's harder to bring them closer to the students in some way that look interesting for them, attractive. (ULA- Jacinta V)
	MT_CONF	This code is used when their claims let you realize that they feel confident about teaching maths	It's what I love most [to teach math]...But when teaching math I'll feel that I'm teaching what I really want to teach. (ULA- Laura A)
	MT_INTEGRATED	This code is used when they claim that maths should be taught as an integrated subject in the sense that every content is needed for learning future ones. Not like other subjects where you can learn a unit and then learn the next one even when they are not related	Any peculiarity of math? In math if you don't learn from the beginning, you won't learn. I think that's what happened to me. In other subjects you can learn a unit and then start the next one. But no in math, because the contents evolve. Every content needs from the previous one. (UFT- Valentina R)

		MT_MECHANIC	This code is used when they claim that for teaching maths the process/mistakes should be more emphasized than the result itself in order to improved learning and understading or that teachers usually do not allow different processes to get to the same result or that they do not pay attention to the process only to the result; or that math is usually taught in a mechanic/theoric way without emphasising thinking, understanding, habilities, application or transference	Many times in mathematics what they teach us is a list of contents and some formulas as the only way you can solve certain problems...like, what you most frequently see promoted in the [mathematics] classroom, it's to try to solve a problem by using a formula, an algorithm (...) but what about the analysis, what about to evaluate different methods, that's not promoted very often in the classroom, the most common thing is to try to solve a formula and that's it. (UCINF- Nicole B 2)
		MT_NOCONF	This code is used when their claims let you realize that they do not feel confident about teaching maths	...nervous, a bit anxious (...) because I'm not good at math. (UFT- Francisca M)
		MT_USEFUL	This code is used when they claim that when teaching Mathematics teachers should help students to realize how math is present in our daily life	...Math is usually taught centered inside the classroom and not related to the utility that mathematical concepts have in our life. Because they are taught as matematical concepts and not daily life concepts, students hate math. But the truth is that mathematics comes from real life. Math wasn't invented for no reason, it was invented because we needed it to solve some problem. I think we need to change that in order that students don't heat math. (ULA- Laura A)
	EXPERIENCE	ME_BAD	This code is used when they claim they have had bad experiences with Mathematics at school or at university	What happened to me...once I was blocked with math. I was in second grade and the teacher gave me an E even when I put a lot of effort on it. All my classmates got an A or B. I remember that the teacher suggested that I was a bit dumb at math. That was when I put math in my blacklist and I failed math ever since. Therefore, I think those are the kind of bad experiences children have during their childhood. (UCINF- Karen G)

		ME_GOOD	This code is used when they claim they have had a good experiences with Mathematics at school or at university	I hated math when I was in highschool, but then, at the University they showed me that math can also be fun. They started to show me math as more friendly, more familiar and that you can play a lot of games using math, and that games teach you a lot. And that, in some sense, cheered me on. (UFT- Francisca M)
LEARNING	VIEW	LV_ACTIVE	This code is used when they say that students should play an active role in their learning process by discovering and/or constructing their own learning	...discovery, that is, (...) you always have to look the way children infer their own learning, that they discover...I don't know...discover the meaning of the words, everything has to be active learning for children, they are constantly looking for ways to solve things... (ULA-Macarena M)
		LV_CONCRETE	This code is used when they refer to learning as being concrete in the sense, that it should start with something familiar to their students, previous knowledge or in contact with their own reality or experience because it takes place faster or is more meaningful	...[Learning] should be much more playful. At schools that have more resources, they learn using concrete material...I don't know...mathematics with cubes for example, and that's how children understand what it's a hundred, a unit... (UFT-Trinidad L)
		LV_EMOTIONS	This code is used when they claim that a key for learning is promoting possitive emotions and motivation in students, p.e. if a student is sad he will not be able to learn	I think that, as a teacher, you have to be dedicated to your students even if you have a big class. You have to dedicate...you have to know what are the difficulties of every student, what they lack. Because sometimes their difficulties are related to their families, because they have problems in them and that's why they can't concentrate during your classes. I think that if you introduce a quote of humanity with your students, you'll develop good methodologies in order to promote their learning. (UCINF-Nicole B)



TEACHING	FEELINGS	TF_CONF	This code is used when they claim that they feel confident about teaching in general	[I feel] fine, because one of my strengths is my creativity so if I see that my methodology is not working, I can change it easily. Therefore, I feel that I'll be able to teach them (UCINF-Karen)
		TF_NOCONF	This code is used when they claim that they do not feel confident about teaching in general	I feel, I'm a bit scared, because you realize that you don't know everything and maybe you don't know anything. So I feel afraid of facing students who might catch me up (UFT-Karina)
	VIEW	TV_CHALLENGE	This code is used when they claim that what teachers should do is to challenge students to make them think and learn meaningfully; or that teachers should not just present contents	Well...it's not just to present contents, otherwise students would learn by themselves because contents are all over the internet. That's why we need teachers, we need teachers that give students adequate materials in order to make them think, not just to memorize the steps but to make them think, pose them challenges (...) That's what I think is really important of being a teacher, to help students to learn. (UFT-Valentina R)
		TV_CONTRADICTION	This code is used when you can see that there are contradictions regarding their teaching views, in the sense that they claim being constructivists but at the same time make some claims that reflect a different view about learning (p.e. behaviorist)	Generalizing, I'd say that most schools are behaviorists (...) and you could say that is more beneficial in terms of knowledge because is faster, easier and everything else. But the problem is that students don't learn how to build their own knowledge, they don't become independent persons. (UCINF-Karen)
		TV_EXPECT	This code is used when they claim that the most important thing in the teaching process is teachers high expectations about students' learning	Based on my internships I've learned that the most important thing is what the teacher expects from their students. Some teachers have told me: "don't waste your time with him, he's not able to do it". In contrast, if the teacher is hopeful about a student, that student will be able to move on. (UFT- Trinidad L)

		TV_MECHANIC	This code is used when they claim that teachers usually teach in a mechanic way (learning by rote) without emphasising understanding or habilities	To expect that students solve a problem in only one way...there are many teachers that claim that there is only one way to divide, I don't know how many there are, I know just two, and they only teach in that way because is the only way the know and like. But students only memorize the steps and they might not even understand why are they dividing or why is division for. In that case you could think that they learned division because they got an A in the test but did they really learn to divide or just memorized the steps? For this reason, your learning objectives always have to be expressed in terms of skills, abilities. (UFT- Valentina R)
		TV_MEDIATOR	This code is used when they claim that the role a teacher should play is a mediator of students learning	As a teacher, you have to be a mediator of your students learning. You have to be there to pose them a problem, to teach them the alphabet, to teach them how to sum, to go along with them during their process, but they are the ones who have to discover the final result. (ULA- Antonia B)
		TV_STUDENT	This code is used when they claim that the role teachers should play in students learning is to identify their needs and their strenghs for improving learning	Well, I think that the teacher always have to be attentive to their students' needs. The teacher has to see and identify the students who have special learning needs, support them... (ULA- Macarena M)
	THINKING VIEW	THV_APPLIED	This code is used when they claim that thinking is being able to apply previous knowledge in new or real life contexts/situations	I understand thinking as the ability to abstract knowledge from different contexts and make them own...thinking is to know how to integrate all my knowledge into my behaviour (...) because, at the end, knowledge is useless if you don't apply it in your everyday life. (ULA- Jacinta V)

		THV_COMP LICATED	This code is used when they say that is difficult to define thinking and they are not able to give any kind of explanation even using their own words	I think that [thinking] is to take the knowledge to something more abstract, or more concrete...I don't know...like to reason...Do you know what I mean? (UFT- Trinidad L 2)
		THV_KNO WLEDGE	This code is used when their claim that your thinking ability is related to the amount of knowledge you have and the ability to relate different concepts	To me the intelligence depends on knowledge, so if you have less knowledge, you're thinking less and you have less abilities (UCINF- Claudio)
		THV_MIND	This code is used when their claim let you infer that they view thinking as the same thing as mind/intelligence or, in other words, everything that takes place in your head	Everything that is going on in my mind (...) that's thinking for me. (UFT- Francisca M)
	TEACHING	THT_CONF	This code is used when they claim that they feel confident about promoting thinking skills in their students in the future	[I feel] fine. I think I can do it, specially divergent [thinking], it's like a strength. (UCINF- Karen)
		THT_LACK	This code is used when they claim that teachers/schools do not encourage students to think	Usually,regrettably, you find this in many schools: students don't think and they only learn how to follow instructions. (...) I think is crucial that students learn how to think by themselves. (...) This might happen in some schools, but not in most of them. (UCINF- Karen G 2)
		THT_NOCO NF	This code is used when they claim that they do not feel confident about promoting thinking skills in their students in the future	It's much harder to make someone think and it's also not so easy to realize if that person is really thinking, specially if you have 25 or 40 students in your class. (UFT- Valentina R 2)
		THT_RELEV ANT	This code is used when they claim that teaching thinking is crucial for long lasting, transferable, applied and/or meaningful learning	I think that [thinking] is crucial. I feel that you don't learn if you're not thinking in the content you're learning. Many times the content is presented and you can memorize it but then it's gone. When you really think, that knowledge is longlasting. (ULA- Jacinta V)

## K. Science Reasoning Task II: Questions reasoning level

**Table 1: Volume and Heaviness questions and classification in terms of reasoning level**

Question	Classification	Reasoning level
1	2A	Early concrete
2	2A	Early concrete
3a	2A	Early concrete
3b	2B	Mature concrete
4		
5	2B	Mature concrete
6	2B	Mature concrete
7	2B	Mature concrete
8	2B	Mature concrete
9	2B	Mature concrete
10	2B/3A	Early formal
11	2B	Mature concrete
12	3A	Mature formal
13a	2B/3A	Early formal
13b	3A	Mature formal
14	3A	Mature formal

## L. Science Reasoning Task II: Scoring rules

**Table 2: Volume and Heaviness Task scoring rules**

<i>Read from the top, go down this list until you find a combination which fits the pupil</i>	
At least TWO 3A items right	3A
At least THREE 2B/3A or 3A items right	2B/3A
Only TWO 2B/3A or 3A items right, provided FOUR or more 2B items are right	2B/3A
FIVE or more 2B items right	2B
Any FOUR 2B items or higher right	2A/2B
At least TWO 2A items right, and THREE 2B or higher items right	2A/2B
THREE 2A items right, and TWO 2B or higher items right	2A/2B
Any TWO 2A items right	2A
ONE 2A item, and THREE 2B or higher items	2A
Up to THREE 2B items, and no 2A, <u>or</u> ONE 2A item, and TWO or less 2B items	1

## M. Science Reasoning Task II: Experimental group tests results

**Table 3: Experimental Group Pre and Post tests results**

Participant ID	Pre-Test Reasoning Level	Post-Test Reasoning Level	Pre-Test Right answers	Post-Test Right answers
1	2B/3A	3A↑	11	14↑
2	2B/3A	3A↑	12	14↑
3	2A/2B	2B/3A↑	7	8↑
4	3A	3A	13	15↑
5	2B	2B/3A↑	10	10
6	2B	2B/3A↑	8	12↑
7	2B/3A	3A↑	10	14↑
8	2B/3A	3A↑	12	11↓
9	2B/3A	3A↑	12	13↑
10	2B/3A	2B/3A	10	11↑
11	2B/3A	2B/3A	13	12↓
12	2B/3A	3A↑	11	10↓
13	2B/3A	3A↑	13	13
14	3A	2B/3A↓	13	13
15	2B/3A	2B/3A	11	13↑
16	2A	2A	4	5↑
17	2A/2B	2B/3A↑	7	9↑
18	3A	2B/3A↓	11	12↑
19	2B/3A	3A↑	8	11↑
20	2B/3A	3A↑	9	9
21	2B	2B	8	9↑
22	2B/3A	3A↑	12	10↓
23	2B/3A	2B/3A	9	11↑
24	2B	2B/3A↑	8	9↑
25	2B/3A	2B/3A	11	13↑
26	2B	2B/3A↑	9	11↑

## N. Science Reasoning Task II: Comparison group tests results

**Table 4: Comparison Group Pre and Post tests Reasoning Level results**

<b>Participant ID</b>	<b>Pre-Test Reasoning Level</b>	<b>Post-Test Reasoning Level</b>	<b>Pre-Test Right answers</b>	<b>Post-Test Right answers</b>
101	2B/3A	3A↑	12	14↑
102	3A	2B/3A↓	14	13↓
103	2B/3A	2B/3A	9	10↑
104	2B/3A	3A↑	13	13
105	2B	2B/3A↑	8	10↑
106	2B	2B/3A↑	9	9
107	2A/2B	2B↑	5	8↑
108	2B	2B/3A↑	8	8
109	2B/3A	2B/3A	11	10↓
110	2B	2B/3A↑	8	9↑
111	2A/2B	2A/2B	7	5↓
112	2B	2A/2B↓	11	7↓
113	2B	2B/3A↑	7	9↑
114	2B/3A	2B/3A	8	7↓
115	2B	2B	10	8↓

## O. Pre and Post tests: Correlation between reasoning level and correct responses

**Table 5: Correlation between the pre-test reasoning level and number of right responses**

			Pre-Test reasoning level	Pre-Test number of correct responses
Spearman's rho	Pre-Test reasoning level	Correlation Coefficient	1.000	<b>.794**</b>
		Sig. (2-tailed)	.	<b>.000</b>
		N	41	41
	Pre-Test number of correct responses	Correlation Coefficient	.94**	1.000
		Sig. (2-tailed)	.000	.
		N	41	41

\*\* . Correlation is significant at the 0.01 level (2-tailed).

**Table 6: Correlation between the post-test reasoning level and number of right responses**

			Post-Test reasoning level	Post-Test number of correct responses
Spearman's rho	Post-Test reasoning level	Correlation Coefficient	1.000	<b>.666**</b>
		Sig. (2-tailed)	.	<b>.000</b>
		N	41	41
	Post-Test number of correct responses	Correlation Coefficient	.666**	1.000
		Sig. (2-tailed)	.000	.
		N	41	41

\*\* . Correlation is significant at the 0.01 level (2-tailed).



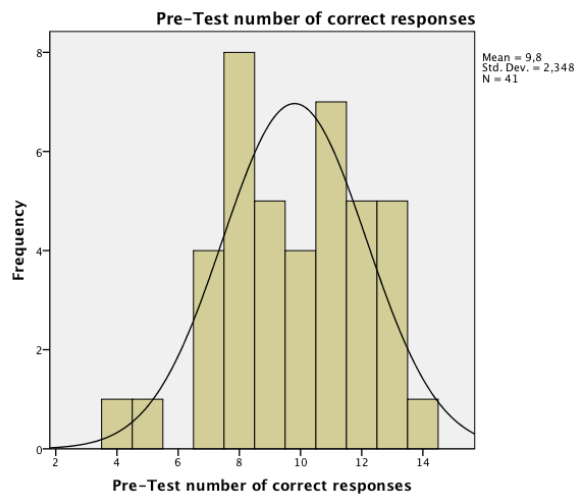
## P. Pre and Post Tests: Tests of normality

**Table 7: Tests of Normality of the number of correct responses for the Pre and Post Tests**

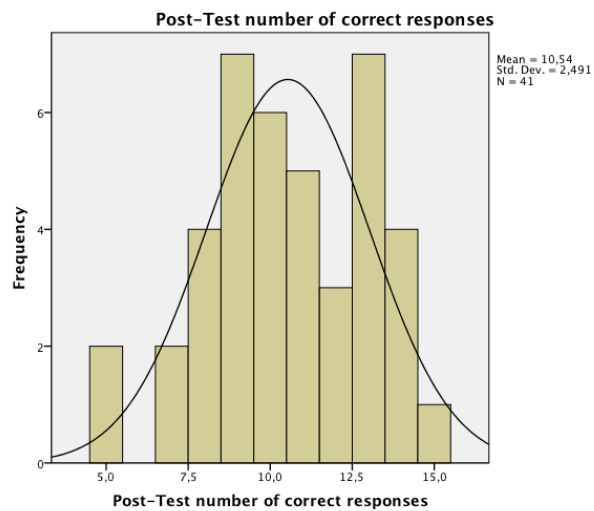
	Tests of Normality					
	Kolmogorov-Smirnov <sup>a</sup>			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Pre-Test number of correct responses	.134	41	.063	.959	41	<b>.142</b>
Post-Test number of correct responses	.131	41	.073	.960	41	<b>.160</b>

## Q. Pre and Post Tests: Histograms of correct responses

**Figure 1: Histogram of number of correct responses on the pre-test**

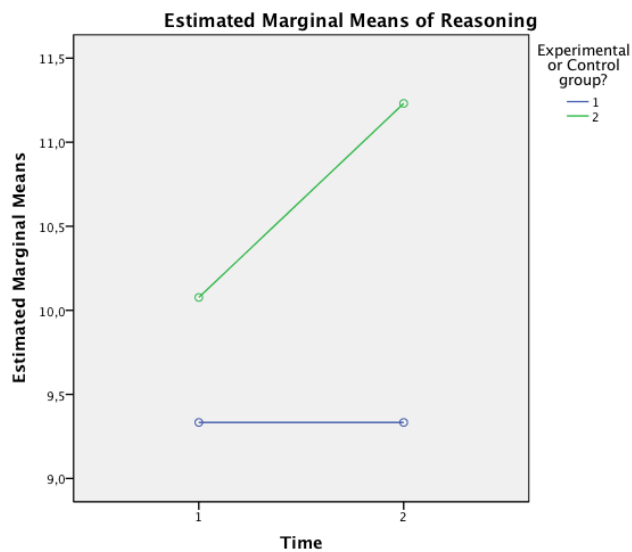


**Figure 2: Histogram of number of right responses on the post-test**



## R. Pre and Post Tests: Variation across time

**Figure 3: Estimated Marginal Means of Reasoning across Time**



The lack of change observed in the comparison group, is represented by Figure 3 that shows the change over time for both groups in terms of average number of right responses. In fact, the comparison group did not change from time 1 (pre-test) to time 2 (post-test) at least in terms of the average number of right responses in the Science Reasoning Tasks test at the beginning and at the end of the term.

## S. Experimental and Comparison Group: Paired sample T test

**Table 8: Paired samples test for the experimental group**

	Paired Differences					t	df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
				Lower	Upper			
Pre-Test number of correct responses - Post-Test number of correct responses	-1.154	1.515	.297	-1.766	-.542	-3.883	25	.001

**Table 9: Paired samples test for the comparison group**

	Paired Differences					t	df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
				Lower	Upper			
Pre-Test number of correct responses - Post-Test number of correct responses	.000	1.890	.488	-1.047	1.047	.000	14	1.000

## T. Pre and Post tests: Independent sample T test

**Table 10: Independent samples test for the pre-test**

		t-test for Equality of Means					
		df	Sig. (2-tailed)	Mean Differ- ence	Std. Error Differ- ence	95% Confidence Interval of the Difference	
						Lower	Upper
Pre-Test number of correct responses	Equal variances assumed	39	<b>.335</b>	-.744	.762	-2.284	.797
	Equal variances not assumed	27.435	.348	-.744	.779	-2.340	.853

**Table 11: Independent samples test for the post-test**

		t-test for Equality of Means						
		t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
							Lower	Upper
Post-Test number of correct responses	Equal variances assumed	<b>-2.499</b>	39	<b>.017</b>	-1.897	.759	-3.434	-.361
	Equal variances not assumed	-2.441	27.326	.021	-1.897	.777	-3.491	-.303

## U. Mann-Whitney U tests

**Table 12: Pre and Post Mann-Whitney tests ranks and statistics**

Group		Mean Rank	Sum of Ranks	Mann-Whitney U	Asymp. Sig. (2-tailed)
Pre-Test reasoning level	Comparison	<b>17.63</b>	264.50	<b>144.500</b>	<b>.140</b>
	Experimental	<b>22.94</b>	596.50		
Post-Test reasoning level	Comparison	<b>16.17</b>	242.50	<b>122.500</b>	<b>.030</b>
	Experimental	<b>23.79</b>	618.50		